Arab Republic of Egypt
Ministry of Education &
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Central Administration of
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Scientific

Student book

Mathematics

Applications

second secondary grade

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Introduction

بسم الله الرحمن الرحيم

Today's world live an age of continuous scientific progress. Tomorrow's generation needs to be well prepared with the materials of the future in order to be able to match with the massive progress of different science. According to this principle, the Ministry Of Education does its best to develop the curricula via placing the learners in the position of being explorer to the scientific truth besides, training the students on the scientific researches as a way of thinking to make the minds the real materials to the scientific thinking and not to be stores for the scientific facts.

We introduce this book " Mathematics Applications" for second secondary grade to be assisting tool to lighten the scientific thoughts of our students and motivate them to search and explore.

In light of what was previously mentioned, the following details have been considered:

★ This book contains three domains: mechanics, geometry and measurements and probability. The book has been divided into related and integrated units. Each unit has an introduction illustrating the learning outcomes, the unit planning guide, and the related key terms. In addition, the unit is divided into lessons where each lesson shows the objective of learning it through the title You will learn. Each lesson starts with the main idea of the lesson content. It is taken into consideration to introduce the content gradually from easy to hard. The lesson includes some activities, which relate Math to other school subjects and the practical life. These activities suit the students' different abilities, consider the individual differences throughout Discover the error to correct some common mistakes of the students, confirm the principle of working together and integrate with the topic. Furthermore, this book contains some issues related to the surrounding environment and how to deal with.

★ Each lesson contains examples starting gradually from easy to hard and containing various levels of thoughts accompanied with some exercises titled Try to solve. Each lesson ends in Exercises that contain various problems related to the concepts and skills that the students learned through the lesson.

★ Each unit ends in Unit summary containing the concepts and the instructions mentioned and General exams containing various problems related to the concepts and skills, which the student learned through the unit.

★ Each unit ends in an Accumulative test to measure some necessary skills to be gained to fulfill the learning outcome of the unit.

★ The book ends in General exams including some concepts and skills, which the student learned throughout the term.

Last but not least. We wish we had done our best to accomplish this work for the benefits of our dear youngsters and our dearest Egypt.

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Second: Geometry and Measurement

Geometry and Measurement

Unit Three

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Probability

Unit Four

Mechanics

Introduction to the development of the science of mechanics

Mechanics, as a general concept, is the science that studies the motion or the balance of bodies through using its own laws; for example there are laws which are applied to the Earth's rotation around the sun and the firing of rockets, a cannon ball or otherwise. It is intended to the change that happens over time to the position of bodies in space. The mutual mechanical effect between bodies is the one by which these bodies change their motions according to the effect of different forces on them. So, the main issue in mechanics is the study of the general laws of the motion and balance of bodies subjected to the action of forces. Mechanics is divided into two branches:

Statics[1]

(**The science of the equilibrium of bodies**) It is concerned with the forces that produce a state of rest in a system of bodies. These forces are known as equilibrium if they don't change the state of the body which is said to be equilibrium under the effect of these forces.

Dynamics[2]

(**The science of the motion of bodies**) It is concerned with the study of forces and their effect on motion of bodies. Dynamics is divided into **Kinematics** which studies motion geometrically (describing motion without reference to the forces causing it), and **Kinetics** which studies the relationship between the motion of bodies and its causes, namely forces.

There are:

Mechanics of Particles (You can ignore the dimensions of the body on studying its motion or equilibrium.)

Mechanics of Rigid Bodies (the body which is consisted of a very large number of connected particles, so close to each other and the distance between any two particles of them is fixed and not affected by any external effect).

Mechanics of Bodies of Variable Mass (Some systems and bodies have varies in which the mass varies as time due to separating out or joining up of particles which decrease or increase during motion. As examples for these bodies, there are the jet rockets and the mining cars; their masses vary as a result of the consumption of fuel and other different systems).

Mechanics of Elastic Bodies (**Elasticity**) It is the property of bodies that are able to return to their original shape and dimensions after being formed; whereas in **plasticity**, if bodies are

1 In this unit, we'll study the concept of force and its properties, its measuring units, resolutions of force into two components and finding the resultant of a set of coplanar forces which act at a point and some applications on that.

2 In this unit, we will study Kinematics which is concerned with the description of the motion of bodies without reference to the forces causing it. This study deals with the motion of bodies and the phenomena associated with this motion and its causes and laws as well as applications on the vertical and horizontal with a uniform acceleration and the general gravitational law of Newton.

exposed to external effects, they don't return to their normal shape on dismissing these effects.

The revelation of mechanics

Classical machanics

It is the oldest branch of mechanics which concerns with the study of forces that act on bodies, It also concerns with the motion of the planets. It helps in many modern technics (constitutive engineering, civil engineering and space's remarks..)

Quantum mechanics

It is a set of physical theories that emerged in the twentieth century, to explain the phenomena at the level of the atom and the particles. It combines between the particle property and the wave length property to show the term of dual wave-particle. Thus, quantum mechanics is responsible for the physical interpretation at the atomic level. Moreover, it is applied on classical mechanics, but its effect doesn't appear at this level. So, quantum mechanics is the generalization of classical physics to be applied on both atomic and normal levels. It is called quantum mechanics due to the importance of quantity in its structure (it is a physical term used to describe the smallest quantity of energy which could be exchanged between particles, and used also to refer to the finite quantity of energy which emits in a discontinuous state.)

Fluid Mechanics

It is a branch of quantum mechanics and it studies mainly fluid (liquids, gases). This branch studies the physical behavior for the fluids and is divided into fluid statics (studying its rest state), and fluid dynamics(studying its motion state).

Biomechanics:

It is the application of the mechanical principles on the living organisms; this includes the study and analysis of the mechanism of living organisms physically, mechanically , physiologically systems. Some simple examples of biomechanics researches include the study of the forces that act on limbs (organs) in its rest or movement status. Some simple examples of that "the movement of the intestine, the blood flowed, the movement of the nucleus in fallopian tube, the transfer of the liquids in the ureter to the kidney and the digested operation of the food and its movement. The Applied Mechanics plays key roles in the study of biomechanics by which we can discover new cases, suitable to improve the applied state.

General relativity theory

The theory of relativity by Einstein changed a lot of concepts with respect to the basic terms in physics, time, place, mass, and energy which brought about a quantum leap in theoretical physics and space physics in the twentieth century. When first published, it modified Newton's mechanical theory that existed for two hundred years. The theory of relativity converted Newton's concept of motion; it stipulates that every motion is relative. The concept of time has been changed from being fixed and definite to yet another non spatial. Time and place has become one thing after being dealt with as two different things. The concept of time is made to depend on the speed of bodies. The dilation and contraction of time has become a key concept for understanding the universe; and so all the Newtonian classical physics have been changed.

Activity

1 - The international web for information (internet) is used to search for the role of mathematicians in improving the science of mechanics. There are some of the searching results:

Thanks to the English scientist **Isaac Newton**, the route of classic mechanics has been prefaced through the laws of motion which illustrated the most of a strological and natural phenomena. The German scientist **Johannes Kepler** and the Italian **Galileo Galilei** have had a great role in putting laws which describe the planets movements.

Kepler's laws show that, there is an attraction power among each of them, and also it shows the movement of planets around the sun according to the new perspective which depends on Heliocentric in a form that calculation in it is matching the astronomical observations substantially. All these rules have been used since the seventeenth century and led to the appearance of the theory of relativity composed by **Einstein** through the years 1905-1916 and the quantum mechanics that composed by the help of **Max plank** , **Heisenberg** , **Schrodinger** and **Dirac** at the beginning of the twentieth century.

Dr. Ahmed Zewail invented a very fast photographic system using laser. It has the ability to determine the motion of the particles when they are formed and when they are connected to each other.

Ahmed Zewail is recorded in the honor list in the United States of America which included **Albert Einstein** and **Alexander Graham Bel**.

For more information search in the Wikipedia using the site ***http: //ar.wikipedia.org***

Measuring Units

When students apply to join the military faculties, some medical tests must be done as height measurment, weight, blood pressure, and average of the beats of the hearts, ...etc.

Measurement operation compares a quantity to another quantity from the same type, to know the number of times the first quantity included into the second quantity.

The system used in most of the parts of the world is called 'International system of units (SI)".

The 'International system of units (SI) include seven basic units. The units of these basic quantities are determined by the direct measurements that depends on the standard units for each of the length, the time, and the weight that was saved in the department of weights and measurements in France. The other units are derivative from the basic units, We are concerned with the following quantities in our study:

First: Fundamental quantities and its measurements units in (SI)

Basic quantities	basic unites	symbols
length	metre	(m)
mass	kilogram	(kg)
time	second	(s)

One of the benefits of using the international system is that: it's too easy to transfer among the units.

Add to your Information:

1- Femtosecond:

is a part of a million billion of a second, i.e. (ten raised to the power of -15) and the ratio between the second and femtosecond is as that between the second and thirty two million years.In 1990, the Egyptian scientist, Ahmed Zweil, was able to install his invention which is known as femto-chemistry, after painstaking effort with his research at California Institute of Technology since 1979. His invention can be summed up in inventing of the unit time surpassed the normal time and access to the femtosecond unit of time.He achieved his scientific discovery using ultra short laser flashes and a molecular beam inside a vacuum chamber, a digital camera with unique specifications so as to photo the motion of the particle since birth and before joining the rest of the other particles.It was then possible to intervene rapidly and surprise the chemical reactions as they occur using the laser flashes as a telescope to watch and follow the destruction and construction processes in the cell. This giant Arab scientist has left the door open to the use of this scientific discovery in the field of medicine, physics, spaces researches and others fields; a new scientific school has been recorded by his name and known as femto-chemistry.

2- The multiples of units:

Unit	symbol	measure
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}

Fractions of units:

units	symbol	measure
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	u	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

According to that we can transfer each of the following units to their corresponding units:

1. 2.75 Km into m.
2. 635 mm. into dm
3. 750 k.Hertz into M.Hertz.
4. 1970 gm into K.gm.

As follow:

1. 2.75 Km =$2.75 \times 1000 = 2750$ m
2. 635 mm. =$635 \times 10^{-2} = 6.35$ dm.
3. 750 k.Hertz= $750 \times 10^{-3} = 0.75$ M.Hertz
4. 1970 gm = $1970 \times 10^{-3} = 1.97$ kgm

Remember that

Km =1000m

M =10 dm

dm = 10 cm

cm = 10 mm

Second: Derived quantities:

1 Unit of measuring the velocity

Velocity is known as the rate of changing of displacement according to time.

Unit of measuring the velocity = unit of measuring the distance÷unit of measuring the time.

So that, velocity is measured by the unit: m/sec. (m/s)

Do you Know

Normative second : is the time interval in which the cesium atom use to ascillate by one complete cycle

2 Acceleration

Acceleration is known as the rate of changing of velocity according to time. So that, acceleration is measured by the unit: m/sec. square (m/s^2)

Note that

The units used for the vector quantities (velocity - Acceleration - force) concerned on their magnitudes regardless of its direction.

According to that we can transfer each of the following units to their corresponding units:

1. 1 km/h into m/sec.
2. 1 Km/h into cm/sec.
3. 1 km/h/sec into m/sec^2
4. 1 km/h/sec into cm/sec^2

Remember that

The average sun day= 24 hour
Hour = 60 min.
Min. = 60 sec.

As follow:

1. $1Km/h = \dfrac{1 \times 1000 \text{ sec}}{60 \times 60 \text{ sec}} = \dfrac{5}{18}$ m/sec

(2) 1 Km/h $= \dfrac{1 \times 1000 \times 100^{m}}{60 \times 60 \text{ sec}} = \dfrac{250}{9}$ cm/sec

(3) km/h/sec $= \dfrac{1000 \text{ m}}{60 \times 60 \text{ sec} \times \text{sec}} = \dfrac{5}{18}$ m/sec^2

(4) Km/h/sec $= \dfrac{1000 \times 100 \text{ m}}{60 \times 60 \text{ sec} \times \text{sec}} = \dfrac{250}{9}$ cm/sec^2

Activity

(1) Transfer each of the following units into their corresponding units:

a 72 km/h into m/sec b 1000 cm/sec into km/h c 36 km/h/sec into cm/sec^2

3 Force

Force is defined as the product of the mass(m) with the acceleration (a)
If we denoted by (F) to the force ,then F= m × a

Units of measuring the magnitude of the force

Absolute units:

As: Dyne and Newton, where: 1 Newton = 10^5 Dyne .and they will define as follow:

Newton: is the magnitude of the force that if it is acts on a mass equals 1 kilogram it gains an acceleration of magnitude 1 m /sec^2

Dyne: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 1 cm /sec^2

Gravitational units:

As: Gram weight (gm.wt) and kilogram weight (Kg.wt), where: 1 Kg.wt = 10^3 gm.wt.

and they will define as follow:

Kilogram weight: is the magnitude of the force that if it is acts on a mass equals 1 Kilogram it gains an acceleration of magnitude 9.8 m /sec^2

Gram weight: is the magnitude of the force that if it is acts on a mass equals 1 gram it gains an acceleration of magnitude 980 cm /sec^2

The Gravitational units joined with **the Absolute units** by the relations: 1 Kg.wt = 9.8 Newton and 1 gm.wt = 980 Dyne

Add to your knowledge

All bodies fell to the ground with uniform acceleration between 9.78. 9.82 m/sec^2 regardless of their masses, Counting on latitude we will consider the acceleration equals 9.8 m/sec^2 for case of use if there is no other values of it are set.

According to that we can transfer each of the following units into their corresponding units:

1. 3.14 Newton into Dyne
2. 6.75×10^7 Dyne into Newton

As follow:

1. 3.14 Newton = 3.14 × 105 = 314000 Dyne
2. 6.75×10^7 Dyne = 6.75×10^7 × 105 = 675 Newton

Activity

2. Transfer each of the following units into their corresponding units:
   - a) $\frac{1}{7}$ gm.wt into Dyne
   - b) 5.36 × 1250 Dyne into Newton
   - c) 2.50 Newton into Dyne

You can put the derived quantities in the following table as follow:

Derived quantity	Relation with other quantities	measurement unit
Velocity (V)	Displacement ÷ time	m/s
Acceleration (a)	velocity ÷ time	m/s^2
Force (F)	mass × acceleration	N

Check your understanding

Choose the correct answer from those given:

1. Mass is measured by:
   - a. Dyne
   - b. Newton
   - c. kilogram
   - d. kilogram weight
2. From the basic quantities in the international system:
   - a. Mass
   - b. Velocity
   - c. Acceleration
   - d. Force
3. Millimeter unit is equivalent to:
   - a. 10^{-3} meter
   - b. 10^{-3} meter cube
   - c. 10^{-2} centimeter
   - d. 10^{-4} decimeter

Answer the following questions

4. Mention the name of the following values
   - a. 10^{-2} meter
   - b. 10^{-3} meter
   - c. 1000 meter
5. Transfer each of the following into meter:
   - a. 63.4 centimeter
   - b. 512.6 millimeter
   - c. 0.534 decimeter
6. **Critical thinking:** Calculate in kilogram unit the mass of water that must fill in a container in a form of cuboid with length 1.6 m ,width 0.650 m and height 36 cm ,known that the density of the water equals 1 gm/cm^3 approximating the result to the nearest integer number.

[Hint: mass = volume × density]

Unit one

Statics

Introduction of the unit

The science of the statics concern with on solving all geometrical problems related to the equilibrium of bodies, the operations of resolving and resultant of the forces acting on a body, and the reaction of the bodies according to the forces act on it, the life applications in houses, buildings, bridges and the designs of engines & machines.

Newton has had more researches and books in this field such as the book , athematical Principles of natural Philosophy which is consisted of three parts and it is the foundation of classical mechanics. One of his famous sayings about himself." I don't know what I may appear to the world but to myself I seem to have been only like a boy playing on the sea-shore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me".

Unit Objectives

By the end of the unit the students should be able to:

- Recognize the concept of a force, force as a vector units of measuring force according to the previous measurments units
- Find the magnitude, the direction of the resultant of two forces act at a point
- Recognize resolution of a given force into two components in a given directions.
- Recognize resolution of a given force into two perpendicular components.
- Find the magnitude and the direction of the resultant of a set forces meeting at a point.
- Investigate equilibrium of a particle under the effect of a set of coplanar forces meeting at a point in the following cases:
  - If two coplanar forces meeting at a point are in equilibrium.
  - If three coplanar forces meeting at a point are in equilibrium.
  - If a set of coplanar forces meeting at a point are in equilibrium.
- Find the resultant of two forces geometrically and algebrically using InformationTechnology as activities
- Recognize the applications of the above study in statics in physical and life situations.

Key - terms

- Statics
- Force
- Rigid body
- Gravitation force
- Acceleration of gravity
- Newton
- Dyne
- Kilogram weight
- Gram weight
- Line of action of the force
- Resolving force
- Force Component
- Equilibrium of a body
- Triangle of forces
- Lami's rule
- Equilibrium of rigid body
- Smooth plane
- Inclined smooth plane
- Centre of gravity

Unit Lessons

lesson (1 - 1):Forces.

lesson (1 - 2):Forces resolution into two components.

lesson (1 - 3):The Resultant of caplanar forces meeting at a point.

lesson (1 - 4): Equilibrium of a rigid body under the effect of coplanar forces meeting at a point.

Materials

- Scientific calculator
- Graphical computer programs

Charts of the unit

1 - 1 Forces

We will learn

- Some basic concepts in statics
- Properties of a force
- Resultant of two forces acting at a point.
- Finding the resultant of two forces acting at a point analytically

Key - term

- Force
- Resultant
- Rigid body
- Gravitation force
- Acceleration of gravity
- Newton
- Dyne
- Kilogram weight
- Gram weight

Materials

- Scientific calculator
- Graphical programs

Preface:

You knew that statics is one of the branches of mechanics that studies forces and conditions of equilibrium of material bodies subjected to acting forces. We will study in this unit only the equilibrium of rigid bodies(1).

Remember that

Scalar quantity is determined completely by a real number (Magnitude) such as distance, time, mass, area, volume.
Vector quantity is determined by its direction in addition to its magnitude
Such as : Force, displacement, velocity, weight, ...

Force

Equilibrium or movement of body depends on the nature of the mechanical mutual influence between it and other objects, i.e on cases of pressure or tension or attraction or repulsion of the body that occur as a result of this influence.

1 Def.

➤ Force : is defined as the effect of a natural body upon another one.

Properties of forces:

The effect of any force depends on the following factors:

First: magnitude of a force.

The magnitude of a force is determined by comparing it by a unit of force, the main units of measuring the magnitude of a force in mechanics are Newton (N) or kilogram weight (kg.wt) **where:**

Add to your knowledge

The natural bodies are divided into:
- **The rigid bodies whose shape does not change under the effect of any force**
- **The elastic bodies whose shape can be reformed under the effect of forces as liquids , gazes , rubber, clay, ...**

➤ 1 Kg.wt = 1000 gm.wt , 1 Newton = 10^5 dyne

➤ 1 Kg.wt = 9.8 newton , 1 gm.wt = 980 dyne

(unless something else is mentioned) (2)

1- **Rigid body: A body whose deformation is neglected whatever the action of the external effect.**

2- **Weight force (weight): it is the gravitational force of attraction between the Earth and the body with acceleration of changes from position to other on the Earth and its approximate magnitude = 9.8m/sec^2 unless something else is mentioned. This topic will be shown in detail elsewhere in mechanics.**

Secondly: Direction of a force

Figure (1) represents the force vector $\overrightarrow{F}$ which could be represented by a directed line segment $\overrightarrow{AB}$ where A is its initial point, B is its terminal point. The magnitude of the force is determined by $\| \overrightarrow{AB} \|$ (its length with a suitable drawing scale). The direction of the arrow is corresponding to the direction of the force $\overrightarrow{F}$, where θ is called the polar angle in the plane of the force $\overrightarrow{F}$. Force is written in the polar form as(f, θ)

Add to your knowledge

Polar angle is the positive angle that the vector makes with the positive direction of x-axis.

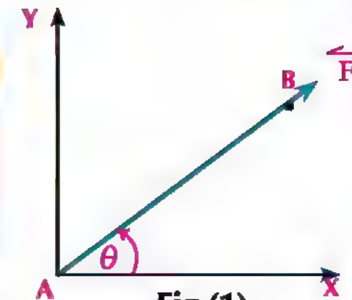


Fig (1)

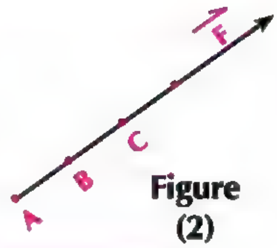


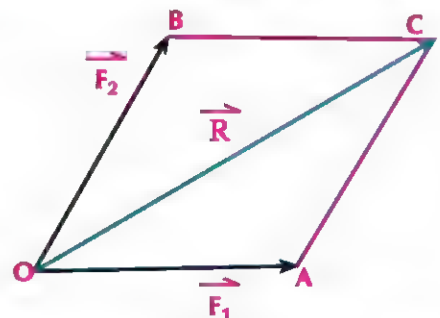
Figure (2)

Thirdly: Point of action of the force and its line of action

In figure (1) : Point A is always coincide on the point of action of the force $\overrightarrow{F}$, it is possible to transfer the point of action of the force to any other point lying on the line of action of the force without changing in its effect on the body as in figure (2), The line of action of the force $\overrightarrow{F}$ in figure (1) is denoted by $\overleftrightarrow{AB}$, i.e. the line of action of a force, is the straight line which passes through its point of action and is parallel to its direction.

Resultant of two forces acting on one point:

for any two forces acting on a body at the same point there is a resultant force $\overrightarrow{R}$ that acts at the same point and has the same effect of the two forces, it is represented geometrically by the diagonal of the parallelogram that represent the two forces by two adjacent sides.

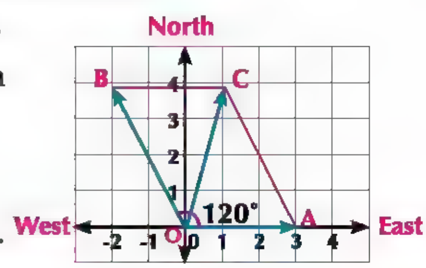


In the opposite figure: $\overrightarrow{R}$ is repreented by the diagonal of the parallelogran ($\overrightarrow{OC}$), $\overrightarrow{R}$ represents the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$. **i.e.:** $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

Activity **Using (GeoGebra) program**

$\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces at a point on a rigid body where $\overrightarrow{F_1}$ = 300 Newton, acts in direction of east, $\overrightarrow{F_2}$ = 400 Newton acts in direction 60° Northwest. Find their resultant.

- Choose a suitable drawing scale (1 cm for each 100 Newton).
- Draw $\overrightarrow{OA}$ to represent the force $\overrightarrow{F_1}$ such that: $\| \overrightarrow{OA} \|$ = 3cm in the positive direction of the x-axis.
- Draw $\angle$ AOB is the polar angle where m ($\angle$ AOB) = 120°
- Draw $\overrightarrow{OB}$ to represent the force $\overrightarrow{F_2}$ such that $\| \overrightarrow{OB} \|$ = 4 cm.
- Draw the parallelogram OACB,

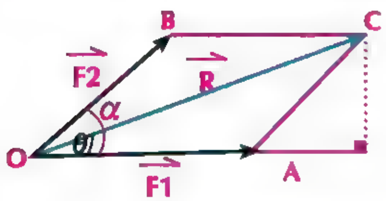


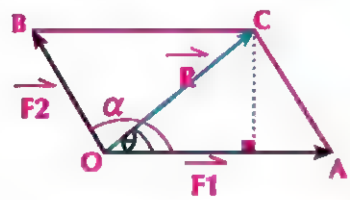
- Notice that the resultant of the two forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ is represented by the directed line segment $\overrightarrow{OC}$
- by using the program we can determine $\| \overrightarrow{OC} \| \simeq$ 3.6 cm. i.e. $R = 3.6 \times 100 \simeq 360$ N
- Notice that: $\overrightarrow{OC}$ inclined by an angle of measure 73° 53\` 53\`\` with $\overrightarrow{OA}$, $\overrightarrow{R}$ makes an angle of inclination of measure 73° 53\` 53\`\` with the direction of $\overrightarrow{F_1}$.

Application on the Activity

1. Use program (*GeoGebra*) to find the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which act on a point an a rigid body where $\overrightarrow{F_1}$ = 400 N acts in the east direction, $\overrightarrow{F_2}$ = 500 N acts in direction 80° north of east.

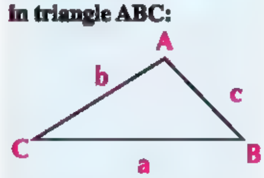
The resultant of two forces meeting at a point analytically





Remember that

Cosine rule:
in triangle ABC:



$a^2 = b^2 + c^2 - 2bc \cos A$

let $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ be two forces acting at O, θ is the angle between their directions. $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are represented by $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{R}$ is represented by $\overrightarrow{OC}$ .let θ be the angle between $\overrightarrow{R}$ and $\overrightarrow{F_1}$, using the cosine rule, we can deduce the magnitude, direction of $\overrightarrow{R}$ as follows:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha} \quad , \quad \tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha}$$

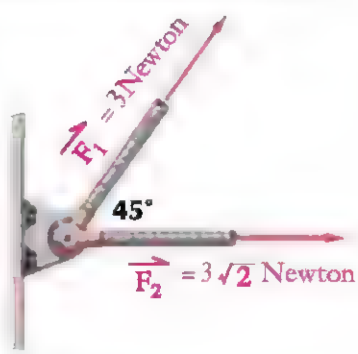
where: F_1 , F_2 , R are the magnitudes of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{R}$ respectively.

Think: How can you investigate the truth of the previous relations ?

Example

1. Two forces of magnitudes 3, $3\sqrt{2}$ newton act on a particle and the measure of angle between their lines of action is 45°. find the magnitude of their resultant and the measure of its inclination angle with the first force.

Solution



let: $F_1 = 3$, $F_2 = 3\sqrt{2}$, $\alpha = 45°$

$\because \quad R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$

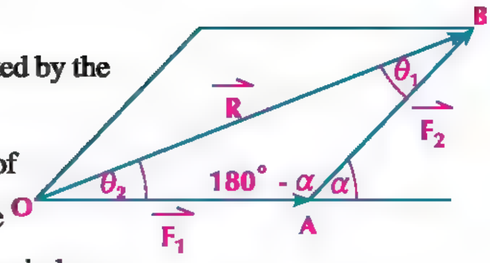
$\therefore \quad R = \sqrt{(3)^2 + (3\sqrt{2})^2 + 2 \times 3 \times 3\sqrt{2}\cos 45°}$

$= \sqrt{9 + 18 + 18\sqrt{2} \times \frac{1}{\sqrt{2}}} = \sqrt{45} = 3\sqrt{5}$ Newton

$\because \tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$ $\quad \therefore \tan\theta = \frac{3\sqrt{2} \times \sin 45°}{3 + 3\sqrt{2}\cos 45°} = \frac{1}{2}$

Using the calculator : m($\angle\theta$) = 26° 33\` 54\`\`

Another solution for the second part of the example :



Notice that: the opposite figure $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are represented by the triangle OAB

where $\angle\theta_1$ is the angle of inclination of line of action of $\overrightarrow{F_2}$ with the resultant $\overrightarrow{R}$, $\angle\theta_2$ is the inclination angle of line of action of $\overrightarrow{F_1}$ with the resultant $\overrightarrow{R}$. using the sin law

Remember that: $\sin(180° - \alpha) = \sin\alpha$;

then: $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{R}{\sin\alpha}$ where $\alpha = \theta_1 + \theta_2$

This rule is used to find the measure of the inclination angle of the resultant $\overrightarrow{R}$ with each of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$

from the previous example:

To find the measure of the inclination angle of $\overrightarrow{R}$ with $\overrightarrow{F_1}$ we use the rule: $\dfrac{F_2}{\sin\theta_2} = \dfrac{R}{\sin\alpha}$

$\therefore \dfrac{3\sqrt{2}}{\sin\theta_2} = \dfrac{3\sqrt{5}}{\sin 45}$

$\therefore \sin\theta_2 = \dfrac{3\sqrt{2} \times \sin 45°}{3\sqrt{5}}$

The measure of the inclination angle of $\overrightarrow{R}$ with $\overrightarrow{F_1}$ equals 26° 33` 54`` as the same result as we get before.

Notice : we can use this method in solving exercices.

Try To Solve

(2) Two forces of magnitudes 10, 6 newton act on a particle and the measure of the angle between their directions is 60°. Find the magnitude of their resultant, and its angle of inclination with the first force.

Critical thinking: Find the magnitude and the direction of the resultant of two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in the following cases:

1- The two forces are perpendicular.

2- The two forces have the same magnitude.

Example

(2) Find the magnitude, and the direction of the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in each of the following cases:

A $F_1 = 5$ newton , $F_2 = 12$ Newton and the angle between their lines of action is 90°

B $F_1 = F_2 = 16$ newton, the measure of angle between their directions equals 120°

Remember that

If $\overrightarrow{F_1} \perp \overrightarrow{F_2}$ then :

$R = \sqrt{F_1^2 + F_2^2}$

$\tan\theta = \dfrac{F_2}{F_1}$

Solution

A $\because \overrightarrow{F_1}$, $\overrightarrow{F_2}$ are perpendicular, then $m(\angle\alpha) = 90°$, then: $\sin(\alpha) = 1$, $\cos(\alpha) = 0$

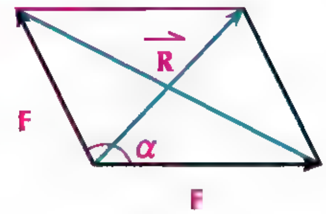
$\therefore R = \sqrt{F_1^2 + F_2^2}$, $\therefore R = \sqrt{(5)^2 + (12)^2} = 13$ Newton

let θ be the measure of the angle between $\overrightarrow{R}$, $\overrightarrow{F_1}$: $\tan\theta = \frac{F_2}{F_1}$ $\therefore \tan\theta = \frac{12}{5}$

$\therefore \theta = \tan^{-1}(\frac{12}{5}) = 67^\circ\ 22`\ 49``$ $\therefore$ the inclination angle of $\overrightarrow{R}$ on $\overrightarrow{F}$ equals $67^\circ\ 22`\ 49``$

B $\because R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$ substitute by $F_1 = F_2 = 16$

$\therefore R = \sqrt{(16)^2 + (16)^2 + 2\times16\times16\cos 120} = 16$ N



we notice that: $F_1 = F_2 = R = 16$ N and the resultant force bisects the angle between the two equal forces i.e. $\overrightarrow{R}$ inclines by an angle of measure 60° with the line of action of each force.

Notice that: From the geometry of the figure : $\cos\frac{\alpha}{2} = \frac{\frac{1}{2}R}{F}$ $R = 2F\cos\frac{\alpha}{2}$

Try To Solve

(3) Find the magnitude and the direction of the resultant of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in each of the following cases:

A $F_1 = 4.5$ newton, $F_2 = 6$ newton and the measure of the angle between them equals 90°

B $F_1 = F_2 = 12$ newton, the measure of the angle between their lines of action equals 60°

Special cases:

1- If the two forces have the same line of action and the same direction:

➢ In this case: $m(\angle\alpha) = 0$; $\cos\alpha = 1$ and by substitution in the resultant rule, we will find that: $R = F_1 + F_2$, the direction of the resultant is the same direction as the two forces, In this case R is called the maximum magnitude of the resultant.

2- If the two forces have the same line of action and opposite directions:

➢ In this case: $m(\angle\alpha) = 180^\circ$; $\cos\alpha = -1$ and by substitution in the resultant rule, we will find that: $R = |F_1 - F_2|$ and $\overrightarrow{R}$ acts in direction of the force or large magnitude, R is called the minimum magnitude of the resultant.

Example: Find the magnitudes of the minimum and the maximum magnitude of the resultant of the two forces of magnitudes 4 , 7 newtons.

➢ Maximum magnitude = $R_{Max} = 4 + 7 = 11$ newton acts in the direction of the two forces.

➢ Minimum magnitude = $R_{Min} = |4 - 7| = 3$ newton acts in direction of force of magnitude 7 newton.

Example

(3) Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120°, the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of $\overrightarrow{F}$ and the measure of the angle that $\overrightarrow{R}$ form with $\overrightarrow{F}$.

Solution

Substituting by: $F_1 = F$, $F_2 = 4$, $R = 4\sqrt{3}$, $\alpha = 120^\circ$

in the rule : $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha$

$\therefore (4\sqrt{3})^2 = F^2 + (4)^2 + 2 \times F \times 4\cos 120^\circ$

$\therefore F^2 - 4F - 32 = 0$ i.e.: $(F + 4)(F - 8) = 0$, then $F = 8$ newton or $F = -4$ (refused)

To find the angle between $\overrightarrow{F}$, $\overrightarrow{R}$ we will use the rule: $\tan\theta = \dfrac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}$

$\therefore \tan\theta = \dfrac{4 \times \sin 120}{8 + 4 \times \cos 120} = \dfrac{1}{\sqrt{3}}$

$\therefore$ $\overrightarrow{R}$ make an angle of measure 30° with $\overrightarrow{F_1}$

Another solution for the second part:

To find the angle between $\overrightarrow{F}$, $\overrightarrow{R}$ we will use the sine rule: $\dfrac{F_2}{\sin\theta_2} = \dfrac{R}{\sin\theta}$

$\therefore \dfrac{4}{\sin\theta_2} = \dfrac{4\sqrt{3}}{\sin 120}$

$\sin\theta_2 = \dfrac{1}{2}$ by reducing and simplifying

$\therefore$ $\overrightarrow{R}$ makes an angle of measure 30° with $\overrightarrow{F}$

Try To Solve

4. Two forces of magnitudes 6, F Kg.wt act on a particle and the measure of the angle between them is 135°. Find the magnitude of the resultant if the line of action of the resultant make an angle of measure 45° with the line of action of the force whose magnitude is F.

Verbal Expression: Find the resultant of two forces of equal magnitudes if they have the same line of action and act in opposite directions.

Exercises (1 - 1)

Complete the following:

1. The effect of a force on a body is determined by the following: ....
2. The vector of the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ is equal to: ....
3. The maximum value of the resultant of two forces of magnitudes 4 , 6 Newton meeting at a point equals ....
4. The minimum value of the resultant of two forces of magnitudes 5 , 9 Newton meeting at a point equals ....
5. 2 , 3 Newton are two forces, if the angle between them is 60°, then the magnitude of their resultant equals ....

Choose the correct answer from those given:

6. The magnitude of the resultant of the two forces of magnitudes 3 , 5 newton and the measure of the angle between them is 60° equals

A 2 N B 6 N a 7 N D 8 N

7. Two forces of magnitudes 3 , 4 N act on a particle and the magnitude of their resultant is 5 N , then the measure of the angle between them equals

A 30° B 45° C 60° D 90°

8. Two equal forces, the magnitude of each of them is 6 N, the magnitude of their resultant is 6N, then the angle between them equals:

A 30° B 60° C 120° D 150°

9. Two forces of magnitudes 3 , F Newton and the measure of the angle between them is 120°. If their resultant is perpendicular to the first force, so the value of F in Newton is

A 1.5 B 3 C $3\sqrt{3}$ D 6

10. If the two forces 6 , 8 N are perpendicular then the sine of the angle of inclination of their resultant with the first force equals:

A $\frac{3}{5}$ B $\frac{4}{5}$ C $\frac{3}{4}$ D $\frac{4}{3}$

Answer the following questions:

11. Two forces of magnitudes 5 , 10 Newton act on a particle and the measure of the angle between them is 120°. Find the magnitude of their resultant and the measure of the angle made by the resultant with the first force.

12. Two forces of magnitudes 3, $3\sqrt{2}$ kg.wt act on a particle and the measure of the angle between them is 45°. Find the magnitude and the direction of their resultant.

13. Two forces of magnitudes 15 , 8 kg.wt act on a particle. If their resultant equals 13 kg.wt, find the angle between the two forces.

14. Two forces of magnitudes 8 , F Newton act on a particle and measure of the angle between them is 120° . If their resultant is $F\sqrt{3}$ N , find the magnitude of F.

15. Two forces of magnitudes 4 , F Newton act on a particle and the measure of angle between them is 135°, If the direction of their resultant is inclined by an angle of measure 45° on F. Find the magnitude of F.

16. Two forces of magnitudes 4 , F Newton act on a particle and the angle between them is 120°. If their resultant is perpendicular to the first force, find the magnitude of F.

17. Two forces of magnitudes F , $F\sqrt{3}$ Newton act on a particle. If the magnitude of their resultant is 2F Newton. Find the measure of the angle between the two forces.

18. Two forces of magnitudes 12 , 15 Newton act on a particle and the (cosine) of the angle between them equals $\frac{-4}{5}$. Find the magnitude of their resultant and the measure of the angle of inclination of the resultant to the first force.

19. Two forces of same magnitude F kg.wt enclose between them an angle of measure 120°. If the two forces are doubled and the measure of the angle between them became 60°, then the

magnitude of their resultant increases by 11 k.g.wt than the first case . Find the magnitude of F.

20) Two forces of magnitudes 12 , F kg.wt act on a point. The first force acts in direction of east and the second force acts in direction 60° south of the west. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° south of the east.

21) F_1 , F_2 are two forces act on a particle and enclose between them an angle of measure 120° and the magnitude of their resultant is $\sqrt{19}$ N , if the angle between them becomes 60° ,then the magnitude of their resultant becomes 7 newton. Find the value of each of F_1 , F_2.

22) Two forces of magnitudes F , 2F kg.wt act on a point, If the magnitude of the second force is doubled, the magnitude of the first force is increased by 15 kg.wt and the direction of their resultant doesn't change. Find the magnitude of F.

Creative thinking:

23) Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. If the direction of one of them is reversed, then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

24) Two forces of magnitudes K , F and the magnitude of their resultant equals 2K, if the measure of the angle between them equals θ, If the measure of the angle changes and become $(180° - \theta)$ then the magnitude of their resultant will decreased to its half. Find the ratio between K , F.

25) F , 2F are two forces act on a particle and enclose between them an angle of measure α. The magnitude of their resultant equals $\sqrt{5}$ F (m + 1) and if the measure of the angle between them becomes $(90° - \alpha)$, then the magnitude of the resultant will be $\sqrt{5}$ F (m - 1).

Prove that $\tan \alpha = \frac{m-2}{m+2}$

Activity

$\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces meeting at a point and the magnitude of their resultant equals R Newton , if the direction of $\overrightarrow{F_1}$ is reversed, then their resultant becomes R $\sqrt{3}$ newton and in a perpendicular direction to the first resultant. Find the measure of the angle between the two forces.

1- Consider that the measure of the angle between the two forces is α and the measure of the angle between the resultant and F_1 is θ.

2- Find $\tan \theta$ then find $\tan (90 - \theta)$ when the direction of F_2 is reversed.

3- Prove that $F_1 = F_2 = F$ from the previous step.

4- Find by using the law of magnitude of the resultant, the resultant of the two forces F_1, F_2 before and after the direction of F_2 is reversed.

5- Could you investigate that: $\cos \alpha = -\frac{1}{2}$ to find the angle between the two forces? investigate that from the previous relation.

6- Do you have another methods for the solution? Mention one of these methods.

1 - 2 Forces resolution

We will learn

- Resolution of a force into two given directions.
- Resolution of a force into two perpendicular directions.

Key - term

- Force Component
- Triangle of forces
- Centre of gravity

Matrials

- Scientific calculator .
- Computer - graph programs.

Preface:

Resolution of a given force into components, generally means finding a group of forces where the known force represents their resultant. We will only study the resolution of a force into two known directions.

Resolution of a force into two given directions

Figure (1): shows resultant vector $\overrightarrow{R}$ which is required to be resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ in the two directions $\overrightarrow{OA}$, $\overrightarrow{OB}$ which make angles of measure θ_1 , θ_2 respectively with $\overrightarrow{R}$

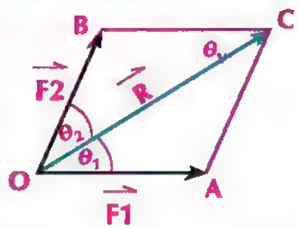


figure (1)

Figure (2): shows triangle of forces where
$\overrightarrow{AC} = \overrightarrow{OB}$

(From the properties of parallelogram) and by applying Sine rule:

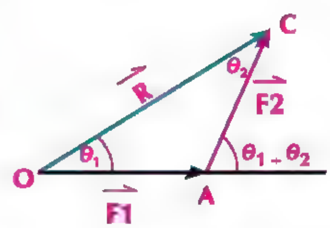


figure (2)

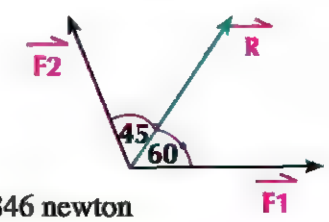
➤ $\dfrac{F_1}{\sin\theta_2} = \dfrac{F_2}{\sin\theta_1} = \dfrac{R}{\sin(\theta_1 + \theta_2)}$

➤ Notice that : $\sin(180 - (\theta_1 + \theta_2)) = \sin(\theta_1 + \theta_2)$

Example

1) Resolve a force of magnitute 12 newton into two components: inclined to the force by angles of measures 60° , 45° in two different sides of it. (Approximate the result into 4 decimal places)

Solution



by applying sine rule:

$$\frac{F_1}{\sin 45^\circ} = \frac{F_2}{\sin 60^\circ} = \frac{12}{\sin 105^\circ}$$

$$\therefore F_1 = \sin 45^\circ \times \frac{12}{\sin 105^\circ} \simeq 8.7846 \text{ newton}$$

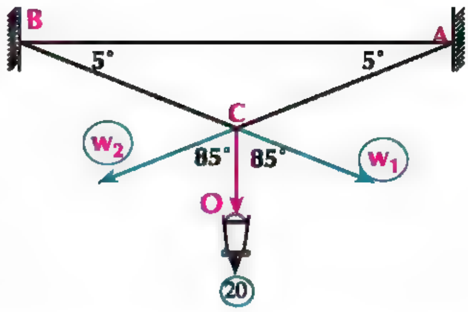
$$F_2 = \sin 60^\circ \times \frac{12}{\sin 105^\circ} \simeq 10.7589 \text{ newton}$$

Try To Solve

1. Resolve a force of magnitude 36 newton into two component inclined to the force by angles of measures 30° , 45° in two opposite sides of the force.

Example **life applications**

2. A lamp of weight 20 newtons suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles, the measure of each is 5°.
   - Resolve the weight of the lamp into two components in the direction $\overrightarrow{AC}$, $\overrightarrow{BC}$ approximating the result to the nearest newton.



Solution

The force of the weight (**20 newton**) is represented by a vector act vertically downwards, starting from the point c, resolve the weight vector into two components as follow:

$$\frac{W_1}{\sin 85^\circ} = \frac{W_2}{\sin 85^\circ} = \frac{20}{\sin 170^\circ} \qquad \textbf{then:}$$

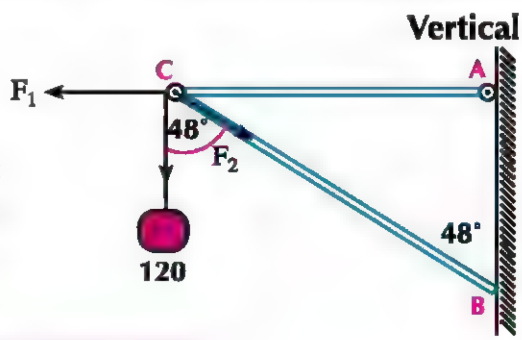
$$W_1 = W_2 = 20 \times \frac{\sin 85^\circ}{\sin 170^\circ} \quad \text{From that we get:}$$

$$W_1 = W_2 = 114.73713 \simeq 115 \text{ newton.}$$

Critical thinking: What happens to the magnitude of the components of the weight in the directions of the two metal rods if the measure of the inclination angle to the horizontal decreated to be smaller than 5°? And what do you expect to the components when the rods become horizontal? Justify your answer.

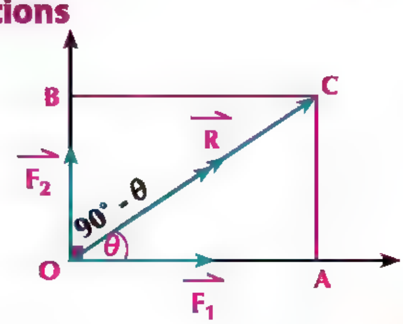
Try To Solve

2. **In the opposite figure:**
   Resolve the vertical force of magnitude 120gm.wt into two components one of them is the horizontal and the other inclined by an angle of measure 48° to the line of action of the force.



Resolution of a force into two perpendicular directions

If $\overrightarrow{R}$ acts on a particle (O) as in the opposite figure and their perpendicular components are $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ where $\overrightarrow{F_1}$ is inclined by an angle of measure θ with $\overrightarrow{R}$, then the parallelogram will be a rectangle ACBO, by applying sine rule on triangle OAC then:

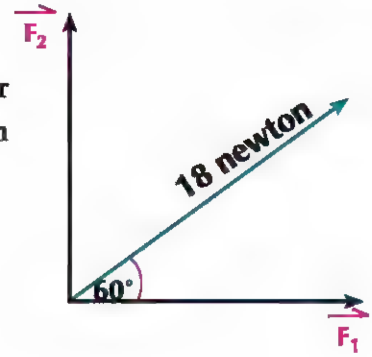


$$\frac{F_1}{\sin(90^\circ - \theta)} = \frac{F_2}{\sin\theta} = \frac{R}{\sin 90^\circ} \qquad \therefore \frac{F_1}{\cos\theta} = \frac{F_2}{\sin\theta} = R$$

and hence we deduce that:

- F_1 (magnitude of the component in a given direction) = $R \cos \theta$
- F_2 (magnitude of the component in the direction perpendicular to the given direction) = $R \sin \theta$

Example



(3) Resolve a force of magnitude 18 newton into two perpendicular components where one of them inclines to the force by an angle of measure 60°.

Solution

$F_1 = 18 \cos 60° = 18 \times \frac{1}{2} = 9$ newton

$F_2 = 18 \sin 60 = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ newton.

Try To Solve

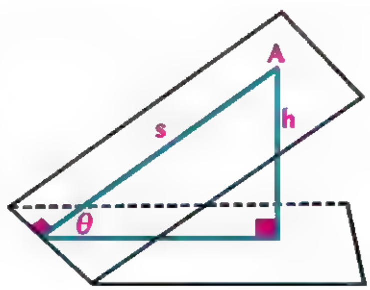
(3) Resolve a force of magnitude $6\sqrt{2}$ newton that acts in the north-east direction into two components one of them acts in the eastern direction, the other in the northern direction.

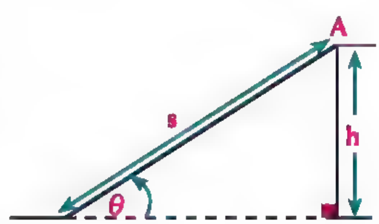
Inclined Plane

It is a surface that inclines to the horizontal plane by and angle whose measure is θ, $0<\theta<\frac{\pi}{2}$ as shown in the opposite figure

The line of the greatest slope is the line lie in the inclined plane orthogonal to the line of intersection of the inclined plane and the horizontal plane the blue line in the given figures, if we denote to its length by (s), height of the inclined plane by (h), angle of inclination to the horizon by (θ), then $\sin \theta = \frac{h}{s}$.

Such that : (h) represents the distance between the Point A and the horizontal, (S) represents the distance between the point A and the line of intersection between the inclined plane and the horizontal plane.





Example

(4) A body of weight 6 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30°. Find the components of the weight in the direction of the line of the greatest slope and the direction normal to it.

Solution

Figure (1): shows the force of the weight of magnitude 6 newton which acts vertically downwards, $\overrightarrow{F_1}$ is the component of the weight in the direction of the line of the greatest slope of the inclined plane downwards, $\overrightarrow{F_2}$ is the

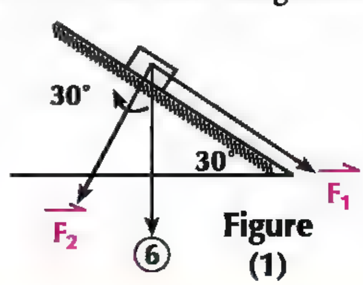


Figure (1)

second component which acts normal to the plane downwards.

($\overrightarrow{F_1}$) : the component of the weight on the direction of the line of the greatest slope.

i.e.: $F_1 = 6 \sin \theta$,

$= 6 \text{ Sin } 30° = 6 \times \frac{1}{2} = 3$ Newton

($\overrightarrow{F_2}$) :the component of the weight acts normal to the plane downwards:

i.e.: $F_2 = 6 \text{ Cos } \theta$

$= 6 \text{ Cos} 30° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ Newton

Verbal Expression: Are the 2 components of force smaller than the force $\overrightarrow{F}$ Itself? Explain your Answer?.

Try To Solve

(4) A rigid body, the magnitude of its weight is 36 newton is placed on a plane inclined to the horizontal at an angle of measure 60°. Find the two components of the weight in a direction parallel to the plane downwards and the direction normal to it.

Add to your knowledge

Rigid body's center of gravity

Is the point where it always the vertical line passing through the suspension point when the body is hanged from any point of it , for example:

(1) The center of gravity of a spherical regular homoginious body is the point where the center of body is located.

(2) The center of gravity of a rod of regular thickness and density is the midpoint of that rod.

Exercises (1-2)

Complete the following:

(1) A force of magnitude 6 Newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East equals .... Newton.

(2) A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components, so its component in the direction of Northern East equals .... Newton.

(3) **In figure (1):**

If the force $\overrightarrow{R}$ is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which make with the force $\overrightarrow{R}$ two angles of measures 30°, 45° from different directions of its line of action, $\|\overrightarrow{R}\| = 12$ newton,

So: $F_1 =$ .... Newton , $F_2 =$ .... Newton.

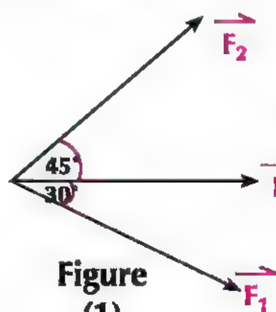


Figure (1)

(4) **In figure (2):**

If the force $\overrightarrow{R}$ is resolved into two components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which make with the force $\overrightarrow{R}$ two angles of measure 45°, 90° from different directions of its line of action and $\|\overrightarrow{R}\| = 18$ Newton, So: $F_1 =$ .... Newton, $F_2 =$ .... Newton

Figure (2)

(5) **In figure (3):**

If the force $\overrightarrow{F}$ is resolved into two perpendicular components $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and the force vector $\overrightarrow{F}$ bisects the angle between the directions of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\|\overrightarrow{F}\| = 6\sqrt{2}$ kg. wt

so: $\|\overrightarrow{F_1}\| =$ .... kg wt ,

$\|\overrightarrow{F_2}\| =$ .... kg wt.

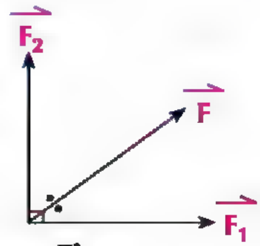


Figure (3)

(6) **In figure (4):**

Force of magnitude $12\sqrt{2}$ newton acts in direction 30° North of the west.

- Magnitude of the component of the force in the western direction = ... Newton.
- Magnitude of the component of the force in the northern direction = ... Newton.

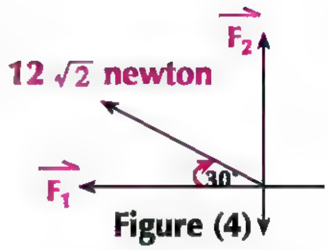


Figure (4)

(7) A force of magnitude 600 gm.wt acts on a particle. Find its two components in two directions making with the force two angles of measures 30° , 45°.

(8) A force of magnitude 120 newton acts in direction of the Northeast. Find its two components in the direction of East and in the direction of North.

(9) Resolve a horizontal force of magnitude 160 gm.wt in two perpendicular directions one of them inclined to the horizontal with an angle of measure 30° upwards.

(10) A force of magnitude 18 newton acts in the direction of South. Find its two components in the 2 directions , 60° East of the South and the other direction towards 30° West of the South.

(11) A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with a angle of measure 60°. Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it.

Creative thinking:

(12) An inclined plane of length 130 cm and height 50 cm, a rigid body of weight 390 gm wt. is placed on it. Find the two components of the weight in the direction of the line of the greatest slope of the plane and the direction normal to it.

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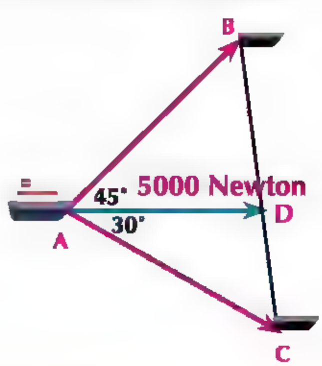
(13) A cruiser is pulled by two ships B and C using two strings hanged to a point A on the cruiser ,the angle between the two strings equals 75° , if the angle between one of the strings and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 Newton and acts on $\overrightarrow{AD}$ Find the tension in the two strands.



The resultant of coplanar forces meeting at a point

1 - 3

Think and discuss

We have studied how to find the resultant of two forces acting on a rigid body in a specific point. The resultant is represented geometrically by the diagonal of the parallelogram drawn by the two forces as two adjacent sides on it.

Can you find the resultant of a set of forces meeting at a point geometrically?

Learn

Resultant of a set of coplanar forces act at a point geometrically:

If set of forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$, ..., $\overrightarrow{F_n}$ act on **a particle as shown in figure (1).**

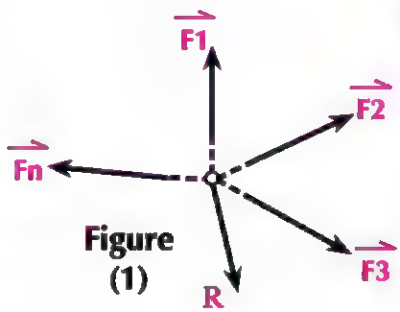


Figure (1)

Using a suitable drawing scale.

draw $\overrightarrow{OA}$ represents $\overrightarrow{F_1}$, $\overrightarrow{AB}$ represents $\overrightarrow{F_2}$, $\overrightarrow{BC}$ represents $\overrightarrow{F_3}$ and so on, ... till $\overrightarrow{F_n}$ is represented by $\overrightarrow{DH}$.

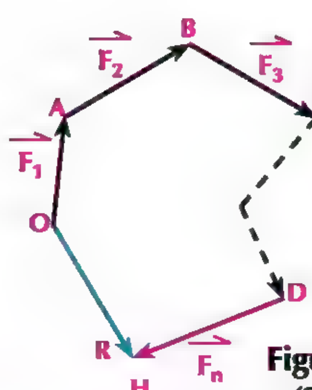


Figure (2)

Then the vector $\overrightarrow{OH}$ in the opposite cyclic order represents the resultant of the forces, where:

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + ... + \overrightarrow{F_n}$

and the polygon is called polygon of forces, it is easy to notice that forming a polygon of forces is the result of applying the triangle of forces several consecutive times.

We will learn

- The resultant of a set of coplanar forces meeting at a geometrical point .
- The resultant of a set of coplanar forces meeting a point analytically.

Key - term

- Resultant
- Algebraic component
- Unit vector

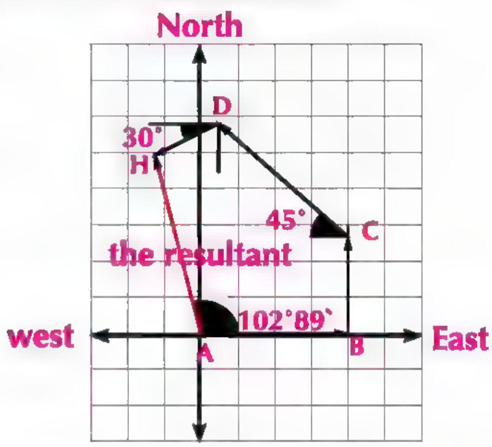
Matrials

- Scientific calculator
- Computer graphics program

Example **Use (GeoGebra) program**

$\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ and $\overrightarrow{F_4}$ are four forces acting at a point on a rigid body where $F_1 = 400$ Newton acts in the Eastern direction, $F_2 = 300$ newton acts in the Northern direction, $F_3 = 500$ newton acts in the Northern-West direction while, $F_4 = 200$ newton, acts in the direction 30° South of the West. Find the resultant of these forces.

1. Draw the directed line segments which represent the forces by scale 1 : 100
2. From the origin draw the vector $\overrightarrow{AB}$ of length 4 units in the Eastern directions.
3. From point B draw the vector $\overrightarrow{BC}$ of length 3 units in the northern direction.
4. From point C draw $\overrightarrow{CD}$ of length 5 units in the (north- west) direction.
5. From point D draw $\overrightarrow{DH}$ of length 2 units in direction 30° south of west.



Remark $\overrightarrow{AH}$ by what you name it?

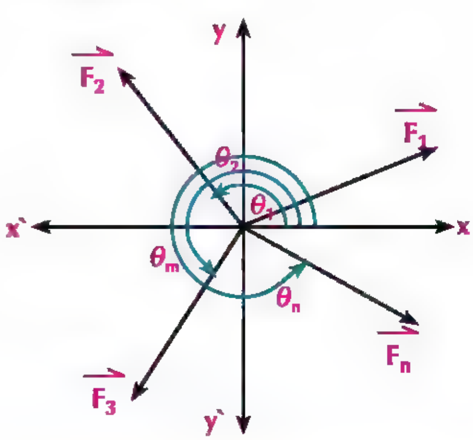
From the graph $\| \overrightarrow{AH} \| = 5.68$ units of length.

$R = 5.68 \times 100 = 568$ newton makes an angle of measure 103° approximatilly.

The resultant of coplanar forces meeting at apoint analytically

If the coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$,...... , $\overrightarrow{F_n}$ act at a point in the coordinate plane system, to make the polar angles θ_1 , θ_2 , θ_3 , ... , θ_n respectively and $\overrightarrow{i}$, $\overrightarrow{j}$ are two fundamental unit vectors in directions $\overrightarrow{OX}$, $\overrightarrow{OY}$ then: $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + ... + \overrightarrow{F_n}$
Resolving each force in the perpendicular directions $\overrightarrow{OX}$, $\overrightarrow{OY}$ then:

$$\overrightarrow{R} = (F_1 \cos\theta_1 \overrightarrow{i} , F_1 \cos\theta_1 \overrightarrow{j})$$
$$+ (F_2 \cos\theta_2 \overrightarrow{i} , F_2 \cos\theta_2 \overrightarrow{j})$$
$$+ \text{.........} + (F_n \cos\theta_n \overrightarrow{i} , F_n \sin\theta_n \overrightarrow{j})$$
$$\overrightarrow{R} = (F_1 \cos\theta_1 + F_2 \cos {}_2\theta + ... + F_n \cos\theta_n) \overrightarrow{i}$$
$$+ (F_1 \sin\theta_1 + F_2 \sin {}_2\theta + ... + F_n \sin\theta_n) \overrightarrow{j}$$
$$\overrightarrow{R} = (\sum_{r=1}^{n} F_r \cos\theta_r) \overrightarrow{i} + (\sum_{r=1}^{n} F_r \sin\theta_r) \overrightarrow{j}$$



- **Where the Expression:** $\sum_{r=1}^{n} F_r \cos\theta_r$ is called the sum of algebraic components of the forces in direction $\overrightarrow{OX}$ and denoted by x.
- **The Expression:** $\sum_{r=1}^{n} F_r \sin\theta_r$ is called the algebraic sum of the components of the forces in direction $\overrightarrow{OY}$ and denoted by y.

The symbol is called segma, Σ denoted for summation and the expression $\sum_{r=1}^{n}$ means the summation of n elements starting from the first element.

Then $\overrightarrow{R} = x\overrightarrow{i} + y\overrightarrow{j}$

and if R is the norm of the resultant , θ is its polar angle then,

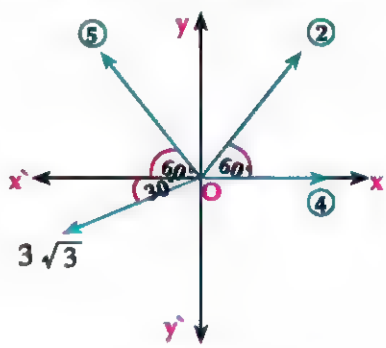
$R = \sqrt{x^2 + y^2}$, $\tan\theta = \frac{y}{x}$

Example

(1) Four coplanar forces act on a particle, the first of magnitude 4 newton, acts in the Eastern direction, The second of magnitude 2 newton, acts in direction 60° North of the East, the third of magnitude 5 newton, acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° west of the south. Find the magnitude and the direction of their resultant.

Solution

Forces of magnitude 4 , 2 , 5 , $3\sqrt{3}$ newton, the measures of their polar angles are zero° , 60° , 120° , 210° respectively. Then we will find the algebraic sum of the components of the forces in direction of $\overrightarrow{OX}$, $\overrightarrow{OY}$



$x = 4\cos 0° + 2\cos 60° + 5\cos 120° + 3\sqrt{3}\cos 210°$

$= 4 + 2 \times \frac{1}{2} - 5 \times \frac{1}{2} - 3\sqrt{3} \times \frac{\sqrt{3}}{2} = 4 + 1 - \frac{5}{2} - \frac{9}{2} = -2$

$y = 4\sin 0° + 2\sin 60° + 5\sin 120° + 3\sqrt{3}\sin 210°$

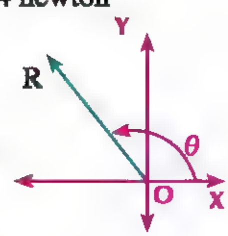
$= 0 + 2 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} - 3\sqrt{3} \times \frac{1}{2}$

$= \sqrt{3} + \frac{5}{2}\sqrt{3} - \frac{3}{2}\sqrt{3} = 2\sqrt{3}$

$\therefore \overrightarrow{R} = 2\sqrt{3}\,\overrightarrow{j} - 2\,\overrightarrow{i}$ $\quad \therefore R = \sqrt{x^2 + y^2} = \sqrt{4 + 12} = \sqrt{16} \therefore R = 4$ newton

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\because x < 0 , y > 0 \quad \therefore \theta = 120°$



meaning that the magnitude of the resultant = 4 newtons, its polar angle is of measure 120°

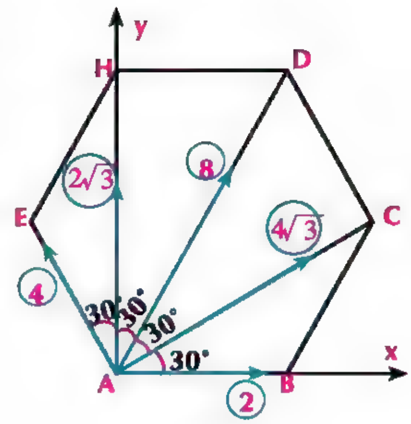
Try To Solve

(1) Coplanar forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 newton act at a point where the measure of the angle between the directions of the first and the second forces= 60° , between the directions of the second and the third = 90° and between the directions of the third and the fourth = 150°. Find the magnitude and the direction of their resultant

Example

(2) A B C D H E is a regular hexagon. Forces of magnitudes 2 , $4\sqrt{3}$, 8 , $2\sqrt{3}$ and 4 kg.wt act at point A in directions $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AH}$, $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

Solution



By considering $\overrightarrow{AB}$ is the direction of the first force so the polar angles to the forces are : 0°, 30°, 60°, 90°, 120° respectively.

$\therefore x = 2\cos 0° + 4\sqrt{3}\cos 30° + 8\cos 60° + 2\sqrt{3}\cos 90° + 4\cos 120°$

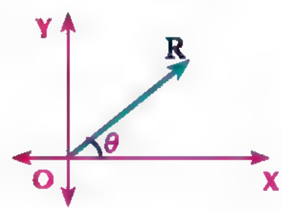
$x = 2 + 4\sqrt{3} \times \frac{\sqrt{3}}{2} + 8 \times \frac{1}{2} + 2\sqrt{3} \times 0 - 4 \times \frac{1}{2}$

$x = 2 + 6 + 4 - 2 = 10$ newton

$y = 2\sin 0° + 4\sqrt{3}\sin 30° + 8\sin 60° + 2\sqrt{3}\sin 90° + 4\sin 120°$

$y = 0 + 4\sqrt{3} \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 2\sqrt{3} + 4 \times \frac{\sqrt{3}}{2}$

$y = 2\sqrt{3} + 4\sqrt{3} + 2\sqrt{3} + 2\sqrt{3} = 10\sqrt{3}$ newton



$\overrightarrow{R} = 10\,\overrightarrow{i} + 10\sqrt{3}\,\overrightarrow{j}$

$\therefore R = \sqrt{x^2 + y^2} = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20$ newton

$\tan\theta = \frac{y}{x} = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\because x > 0 , \ y > 0 \qquad \therefore m(\angle\theta) = 60°$

i.e. The resultant acts in the direction of $\overrightarrow{AD}$

Exercises (1 - 3)

Complete the following:

(1) If the forces $\overrightarrow{F_1} = 2\,\overrightarrow{i}$, $\overrightarrow{F_2} = \overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_3} = 6\,\overrightarrow{j}$ then:
the magnitude of the resultant of the forces = ... and its direction = ...

(2) If the forces $\overrightarrow{F_1} = 2\,\overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_2} = 4\,\overrightarrow{i} - 8\,\overrightarrow{j}$, $\overrightarrow{R} = 2a\,\overrightarrow{i} - 3b\,\overrightarrow{j}$
then: a = ... , b = ...

(3) If $\overrightarrow{F_1} = 3\,\overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} - \overrightarrow{j}$, $\overrightarrow{F_3} = 4\,\overrightarrow{i} - b\,\overrightarrow{j}$, $\overrightarrow{R} = 6\,\overrightarrow{i} - 4\,\overrightarrow{j}$
then: a = ... , b = ...

4. Find the magnitude and the direction of resultant of the forces shown in each of the following figures:

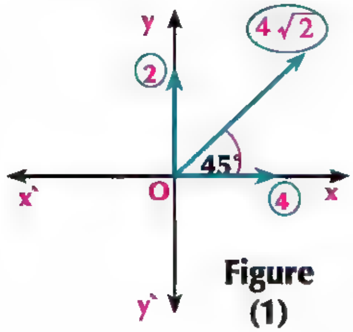


Figure (1)

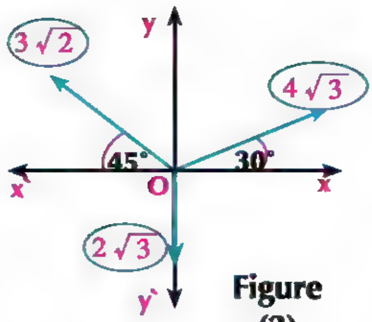


Figure (2)

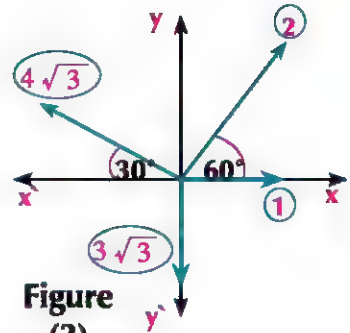
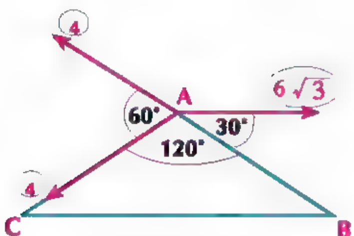
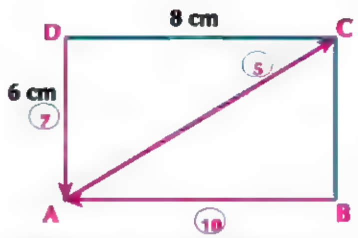


Figure (3)



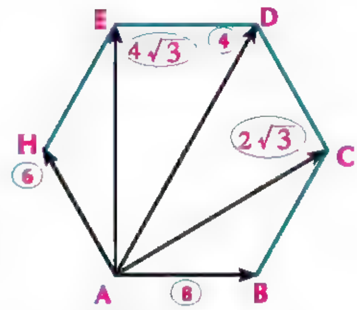
Isosceles triangle

Figure (4)



Rectangle it's dimensions are 6cm , 8cm

Figure (5)



Regular hexagon

Figure (6)

5. The forces 3 , 6 , $9\sqrt{3}$ and 12 kg.wt act on a particle and the measure of the angle between the first and the second is 60° , between the second and the third is 90° and between the third and the fourth is 150°. Find the magnitude and the direction of resultant of these forces.

6. Three forces of magnitudes 10 , 20 , 30 newton act at a particle. The first acts towards the east and the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° South of the west. Find the magnitude and the direction of resultant of these forces.

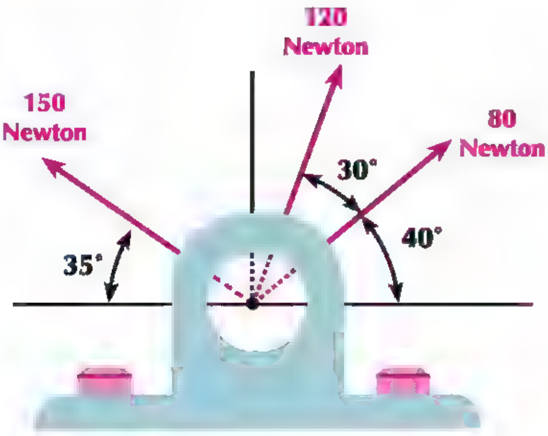
7. Four forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 gm.wt act on a particle, the first acts in the east direction and the second acts in the direction 60° north of the east and the third acts in the direction 30° north of the west and the fourth acts in the direction making an angle of 60° South of the east. Find the magnitude and direction of resultant of these forces.

8. A B C is an equilateral triangle , M is the point of intersection of its medians. The forces of magnitudes 15 , 20 , 25 newton act on a particle in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$. Find the magnitude and the direction of the resultant of these forces.

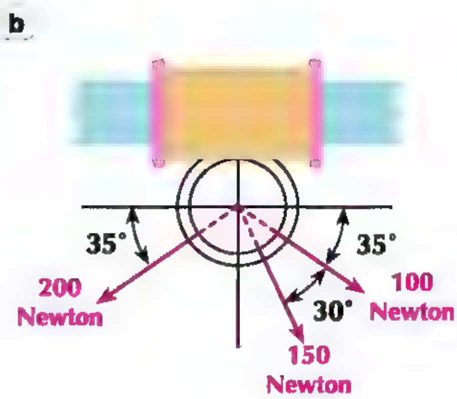
9. ABCD is a square of side length 12cm , $H \in \overline{BC}$ so BH = 5cm. Forces of magnitudes 2 , 13 , $4\sqrt{2}$ and 9 gm.wt act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

(10) From the data represented in the opposite figure Find the magnitude and the direction of the resultant

a

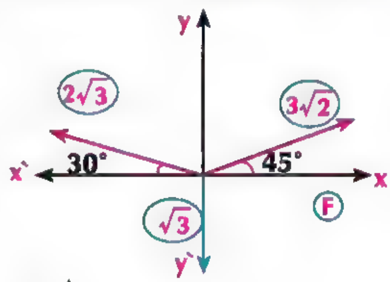


b

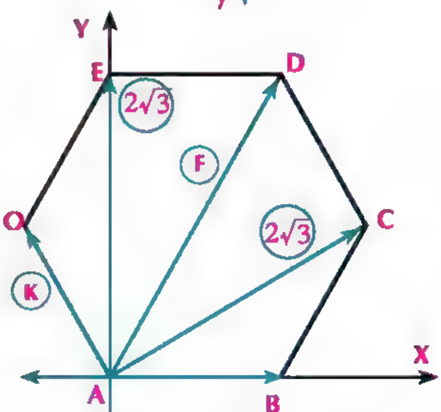


(11) If $\overrightarrow{F_1} = 5\overrightarrow{i} + 3\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} + 6\overrightarrow{j}$ and $\overrightarrow{F_3} = 14\overrightarrow{i} + b\overrightarrow{j}$ are three coplanar forces meeting at a point and their resultant $\overrightarrow{R} = (10\sqrt{2}\ .\ 135°)$ Find the values of a , b

(12) In the opposite figure :
If the magnitude of the resultant of the forces equals $3\sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force

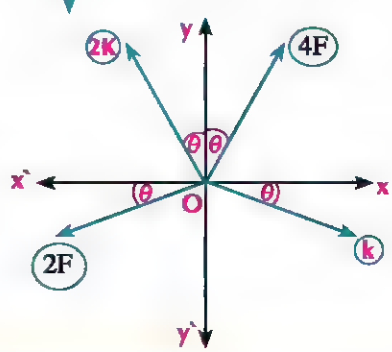


(13) In the opposite figure :
If the magnitude of the resultant of the forces equals 20 Kg.wt and acts in the direction of $\overrightarrow{AD}$ Find the values of F and K.



Creative thinking:

(14) **The opposite figure :** shows four coplanar forces act at the point (O) in the directions shown in the figure, where $\sin\theta = \frac{4}{5}$ and the resultant of these forces equals $8\sqrt{2}$ newton and makes an angle of measure 135° with $\overrightarrow{OX}$, then find the values of F , K.



Equilibrium of a rigid body under the effect of coplanar forces meeting at a point

1 - 4

If two forces or more act on a rigid body and the status of the body does not change, it is said that the two forces or the forces are equilibrium and the body is in an equilibrium state. The simplest type of equilibrium resulted by the effect of two forces on a rigid body.

Equilibrium of a rigid body under the effect of two forces

Co-operative work

1- Put a body of weight 20 kg.wt on the scale pan of a horizontal pressure balance and notice the reading of the balance. figure (1)

2- Ask your classmate to suspend the same body by a light smooth string and the end of the string by a hook of spring balance then to notice the reading of the spring balance in the of case of rest . figure (2)

3- Compare the results in the two experiments. What do you notice?

Notice:

➢ Each of the reaction r in the first and tension force T in the second experiment equals 20 kg.wt which is the weight of the body.

Learn

Terms of balancing a rigid body under the effect of two forces

A rigid body is balanced under the effect of two forces only if these two forces are:

1- equal in magnitude.

2- opposite in direction.

3- their lines of action are on the same straight line.

We will learn

- Equilibrium of a rigid body under the effect of two forces.
- Equilibrium of a rigid body under the effect of three forces meeting at a point.
- Triangle of forces rule
- Lami`s rule
- The three forces theorem
- Equilibrium of a set of coplanar forces act at a point.

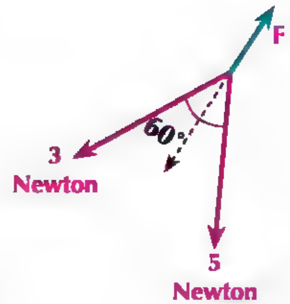
Key - term

- Triangle of forces rule
- Lami`s rule
- Polygon of forces

Matrials

- Scientific calculator
- Computer graphics programs.

Example



1. If a force of magnitude F is in equilibrium with two forces of magnitudes 5 Newton and 3 Newton act at a point and enclosed between them an angle of measure 60°. Find the value of F.

Solution

We can find the resultant of the two forces 5,3 N from the rule:

$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$ $\quad \therefore R = \sqrt{25 + 9 + 2 \times 5 \times 3 \cos 60^\circ}$

$\therefore R = \sqrt{25 + 9 + 15} = \sqrt{49} = 7$ newton

$\because$ the force (F) and the resultant of the two forces (R) are in equilibrium $\therefore F = 7$ newton

Try To Solve

1. If a force of magnitude F is in equilibrium with two perpendicular forces of magnitudes 5 Newton and 12 Newton. Find the magnitude of the force F.

Transfer the point of action of a force to any point on its line of action:

Activity

1. Use the following tools: spring balance – thin metal disk -water balance – ruler
2. Set the table horizontally using the water balance.
3. Join the disk by two light strings at the two holes A , B then join the other two ends of the strings by the spring balance.
4. Set the ring of one of the two balances in a nail fixed on the table at (C) ,then pull the other balance and set it at (D) to another nail far from the other nail so that the two strings are tensioned as in the figure.
5. Find the magnitude of the tension acting in the string and record the results.
6. Change the position of the end of the string from point A to A_1 , A_2, A_3 … also change the position of the end of the other string from point B to B_1 , B_2, B_3 … recognize the reading of the spring balance in each case and record the results …what do you notice ?

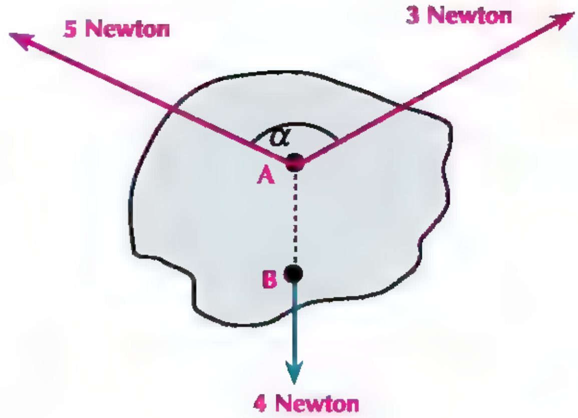
We note that: the two readings are equal in the state of equilibrium.

From the previous activity we deduce that :

If a rigid body is in equilibrium under the effect of two forces, then we can transfer the point of action for any force to another point on its line of action without any impact on the equilibrium of the body.

Example

(2) The forces 3, 4 and 5 Newton are in equilibrium as in the opposite figure Find the measure of the angle between the two forces 3N,5N



Solution

∵ The set of forces are in equilibrium

∴ The resultant of the two forces 3N , 5N is equilibrium with the force 4N. If the measure of the angles between the two forces 3N , 5N is α, then :

$R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha$

By substitution: $R = 4$, $F_1 = 3$, $F_2 = 5$

$16 = 9 + 25 + 2 \times 3 \times 5 \cos\alpha$ ∴ $30 \cos\alpha = -18$

So that $\cos\alpha = \frac{-3}{5}$

∴ $m(\angle\alpha) = 180^\circ - 53^\circ\ 7'\ 49'' = 126^\circ\ 52'\ 11''$

Try To Solve

(2) If the forces 7,8 and 13 Newton are in equilibrium ,Find the measure of the angle between the first and the second two forces .

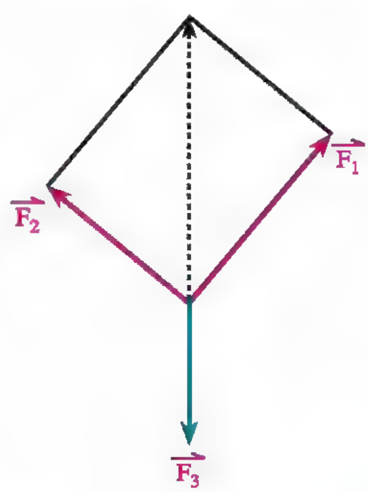
Equilibrium of a rigid body under the action of three coplanar forces meeting at a point

You have studied the necessary and sufficient conditions of the equilibrium of a rigid body under the effect of two forces .Now we will study the equilibrium of three coplanar forces their lines of actions met at a point, these forces either act in a point (or a particle) or act on a body such that their lines of actions met at a point.

Learn

If it is possible to represent three coplanar forces meeting at a point by the sides of triangle of directions taken in the same cyclic order, then the forces are equilibrium.

So, in the opposite figure: The three forces will be in equilibrium if their magnitudes represent the sides length of a triangle taken in the same cyclic order.



Verbal Expression

Show which of the forces whose magnitudes are listed below are equilibrium? Explain your answer.

Consider that the forces are in different direction and act on one point:

(a) 3,5,9 N (b) 3,5,7 N (c) 4,10,6 N

Triangle of forces rule

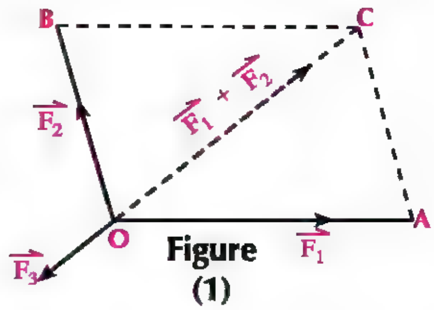


Figure (1)

Figure (1): represents the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ that act at a rigid body in directions $\overrightarrow{OA}$, $\overrightarrow{OB}$.

The resultant of the two forces is ($\overrightarrow{F_1}$ + $\overrightarrow{F_2}$) which act on the diagonal $\overrightarrow{OC}$ of the parallelogram OACB.

But $\overrightarrow{F_3}$ equals in magnitude and opposite in direction to $(\overrightarrow{F_1}+\overrightarrow{F_2})$

$\therefore\ \overrightarrow{F_1}+\overrightarrow{F_2}+\overrightarrow{F_3}=\overrightarrow{0}$ $\qquad\therefore\ \overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$ are equilibrium forces.

Check your understanding:

Show that the forces $\overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$ are equilibrium such that:

$F_1 = 2\overrightarrow{i} - \overrightarrow{j}$, $F_2 = \overrightarrow{i} + 3\overrightarrow{j}$, $F_3 = -3\overrightarrow{i} - 2\overrightarrow{j}$

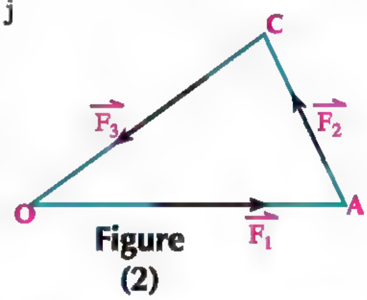


Figure (2)

Figure (2): represents the triangle of forces of three equilibrium coplanar forces $\overrightarrow{F_1}$ ، $\overrightarrow{F_2}$ ، $\overrightarrow{F_3}$, where the lengths of sides of the triangle are in a proportion with the magnitudes of the corresponding forces.

i.e.: $\frac{F_1}{OA}=\frac{F_2}{AC}=\frac{F_3}{CO}$

So that: If three coplanar forces met at a point are equilibrium and a triangle is drawn such that its sides are parallel to the lines of action of the forces and taken the same cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of their corresponding forces

Think: Use the sin rule to prove "Triangle of forces rule".

Example

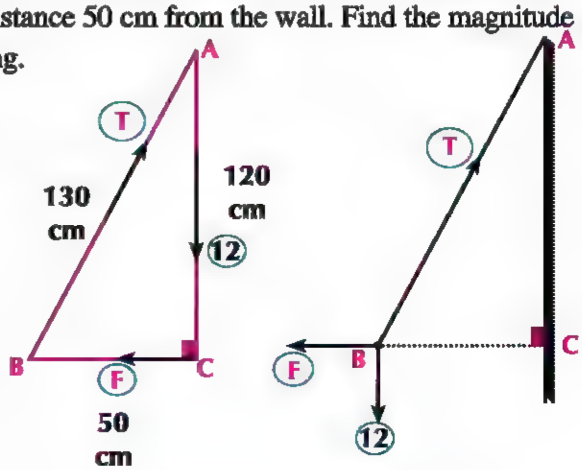
(3) A body of weight 12 Newton is hanged from one end of a light string whose length 130 cm. The other end is fixed at a point in the vertical wall, the body is pulled by a horizontal force which makes the body in equilibrium when it is at distance 50 cm from the wall. Find the magnitude of each of the force and the tension in the string.

Solution

The weight is in equilibrium under the effect of three forces:

- The weight of magnitude 12 N that acts vertically downwards.
- The horizontal force F.
- The tension in the string which acts in the direction $\overrightarrow{BA}$

In Δ BAC We can find the length of $\overline{AC}$ from pythagorus rule.



$AC = \sqrt{(130)^2 - (50)^2} = 120$ cm

Δ BAC is the a triangle of forces: $\frac{T}{130} = \frac{12}{120} = \frac{F}{50}$ T = 13 newton , F = 5 newton

Try To Solve

(3) A weight of 16 Newton is hanged by a string of length 50 cm and the other end of the string is fixed on a point in the ceiling of a room . The weight is pulled by a horizontal force till it becomes equilibrium when it is at a distance 40 cm from the ceiling , find the magnitude of the force and the tension in the string.

lami's rule:

If the coplanar forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ act at a point and equilibrium as in figure (1) ,then it could be represented by the sides of a triangle taken in the same cyclic order as in figure (2)

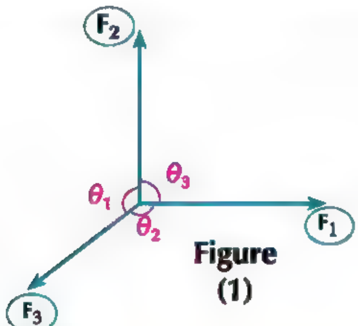


Figure (1)

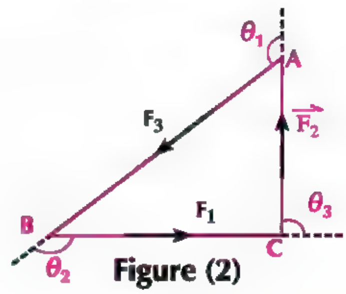


Figure (2)

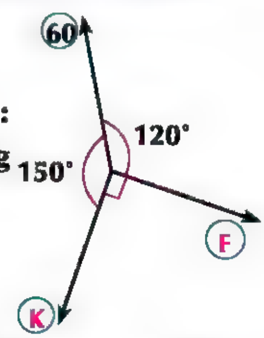
Using the sine law:

$$\frac{BC}{\sin(180 - \theta_1)} = \frac{CA}{\sin(180 - \theta_2)} = \frac{AB}{\sin(180 - \theta_3)} \quad \text{i.e.} \quad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

If a body is in equilibrium under the effect of three forces meeting at a point, then the magnitude of each force is proportional to the sine of the angle between the two other forces

Example

(4) Three coplanar forces of magnitudes 60, f and k Newton meeting at a point are in equilibrium. If the measure of the angle between the line of action of 1st and 2nd forces is 120° and between the 2nd and the 3rd is 90° Find the value of each of f and k.



Solution

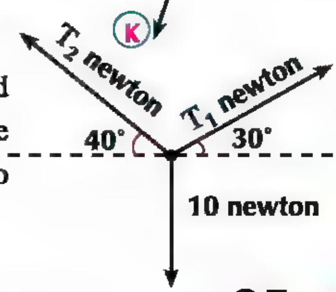
The system is in equilibrium under the effect of the following three forces:

a force of magnitude 60 N , forces of magnitudes F , k N By applying lami's rule we get: $\frac{60}{\sin 90°} = \frac{F}{\sin 150°} = \frac{K}{\sin 120°}$

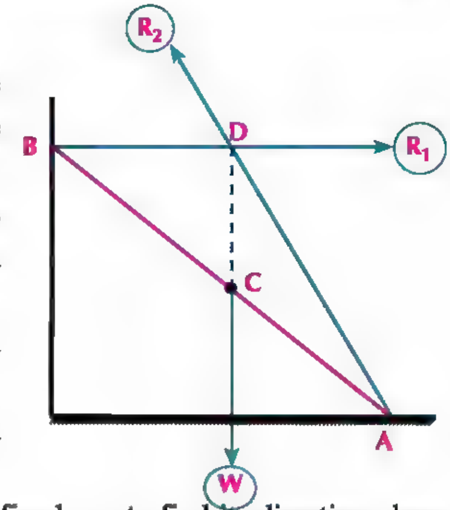
$\frac{60}{1} = 2F = \frac{2K}{\sqrt{3}}$ i.e.: F = 30 newton , $K = 30\sqrt{3}$ newton

Try To Solve

(4) In the opposite figure: A weight of magnitude 10 Newton is suspended by two strings, the first is inclined by an angle of measure 30° to the horizontal, and the second is inclined by an angle of measure 40° to the horizontal. Find T_1 , T_2 in case of equilibrium.



1 - 4 Equilibrium of a rigid body under the effect of coplanar forces meeting at a point



Rule:

If a rigid body is in equilibrium under the effect of three coplanar non parallel forces, then the lines of action of the three forces meet at one point.

For example: If a uniform rod of weight w rests with its end B on a smooth vertical wall and with its end A on rough horizontal ground, then:

- The weight of the rod acts at the midpoint of the rod vertically downwards.
- Reaction of the wall R_1 is perpendicular to the smooth wall in the direction $\overrightarrow{BD}$.
- Reaction of the rough ground R_2, its direction is not defined , so to find its direction, draw $\overrightarrow{AD}$ which passes through the point D (point of intersection of the two lines of action of $\overrightarrow{W}$ and $\overrightarrow{R_1}$) as in the figure.

Example

(5) A metallic smooth regular sphere of weight 1.5 kg .wt. and radius length 25 cm is suspended from a point B on its surface by a string of length 25 cm. Its other end A is fixed at a point in a smooth vertical wall to be in equilibrium as it rests on the wall. Find the magnitude of the tension in the string and the reaction of the wall.

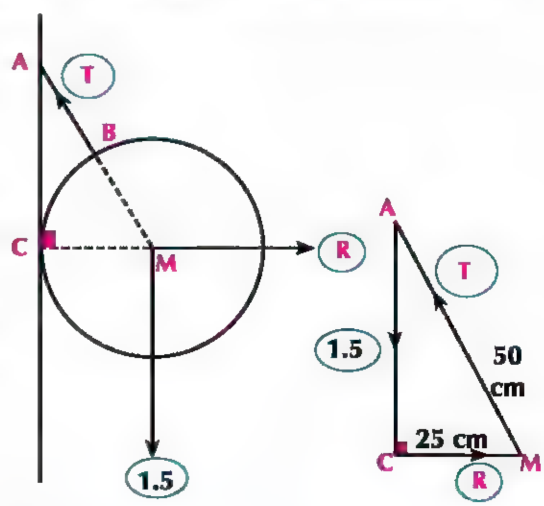
Remember that

The center of graphity of a homaginious sphere is its graphical center

Solution

The sphere is in equilibrium under the effect of three forces:

- The weight of the sphere whose magnitude is 1.5 kg wt. acts vertically downwards .
- The reaction of the wall of magnitude R, acts at the point of touch of the sphere with the wall in a perpendicular direction to the wall, hence it passes through the center of the sphere M.
- The tension in the string of magnitude T acts in the direction of $\overrightarrow{BA}$,



$\because$ The weight force and the reaction force meet at M.

$\therefore$ The line action of the tension force in the string should pass through the point M.

i.e. Δ MAC is the triangle of forces where: MA=MB+BA, MA=25+25=50cm. CM=25cm.

Δ AMC is a right-angled triangle

: $AC = \sqrt{(50)^2 - (25)^2} = 25\sqrt{3}$ cm

Apply triangle of force Rule

$$\frac{T}{50} = \frac{1.5}{25\sqrt{3}} = \frac{R}{25} \qquad \therefore T = \sqrt{3} \text{ kg.wt} , \quad R = \frac{\sqrt{3}}{2} \text{ kg.wt.}$$

Think: Can you solve the previous example using another methods? mention these methods, then use one of them to solve the previous example.

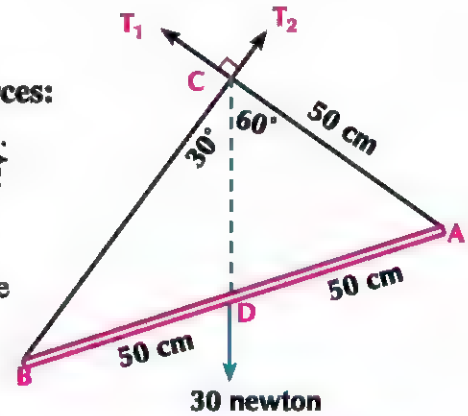
Try To Solve

5) A metallic smooth regular sphere of weight 100 gm.wt and radius length 30 cm is suspended from a point on its surface by a string of length 20 cm. Its other end is fixed at a point in a smooth vertical wall to be in equilibrium. Find the magnitude of the tension in the string and the magnitude of the reaction of the wall.

Example

6) A uniform rod of length 100 cm and weight 30 N is suspended at its ends freely by two perpendicular strings, their two ends are fixed at a hook. If the length of one string is 50 cm. Find the magnitude of the tension in each of the two strings.
When the rod is hanged freely and in equilibrium.

Solution



The rod is in equilibrium under the effect of three forces:
The weight 30 N, act vertically down in the middle of the rod.
The tension in the two strings T_1, T_2 that act on $\overrightarrow{AC}$, $\overrightarrow{BC}$ respectivly and intersected prependiculary at C.

$\because$ $\overrightarrow{CD}$ is drawn from the vertex of the right angle to the midpoint of the hypotenuse.

$\because$ D is the midpoint of the hypotenuse $\overline{AB}$

$\therefore CD = \frac{1}{2} AB = 50\text{cm}$ $\quad\therefore$ ACE is an equilateral triangle

$\therefore m(\angle ACD) = 60^\circ$, $m(\angle BCD) = 30^\circ$

Applying Lami's rule we get:

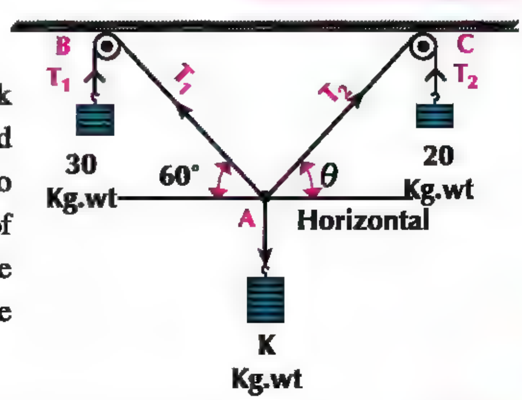
$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{30}{\sin 90^\circ} \qquad T_1 = 15 \text{ newton}, \; T_2 = 15\sqrt{3} \text{ newton}$$

Think: Use another method to solve the previous example.

If a string passes through a smooth pulley and stretched on it, then the two tensions in the two terminals of the string are equal "in magnitude.

Example

7) **In the opposite figure:** A weight of magnitude k is suspended by an end of a string , the other end is suspended by two strings passing over two smooth pulleys at B, C and carries two weights of magnitudes 30 kg.wt and 20 kg.wt Find the value of the weight k and the measure of angle θ in state of equilibrium.



Solution

In the previous figure : Let the tensions in the strings be T_1 , T_2 and act in the directions $\overrightarrow{AB}$, $\overrightarrow{AC}$

$\because$ the pulleys are smooth, then : $T_1 = 30$ kg.wt , $T_2 = 20$ kg.wt

The body of weight K is in equilibrium under the effect of three forces: the weight of magnitude k and the tensions in the strings T_1 , T_2

Applying Lami's rule we get:

$$\frac{30}{\sin(90^\circ + \theta)} = \frac{20}{\sin(60^\circ + 90^\circ)} = \frac{K}{\sin[180^\circ - (60 + \theta^\circ)]} \quad \text{By simplifying}$$

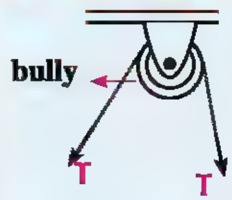
$$\frac{30}{\cos\theta} = 40 = \frac{K}{\sin(60 + \theta^\circ)}$$

$\therefore \cos\theta = \frac{3}{4}$ $\qquad \therefore m(\angle\theta) = 41^\circ\, 24`\, 35``$

$K = 40 \times \sin(41^\circ\, 24`\, 5`` + 60^\circ)$

$\therefore K \simeq 39.2107$ kg.wt

Notice that



The tension is equal in the two parts of the strings

Remember that

$\sin(90^\circ + \theta) = \cos\theta$

$\sin(180^\circ + \theta) = \sin\theta$

Try To Solve

(6) The ball of a pendulum of weight 600 gm wt is displaced until the string makes an angle of measure 30° to the vertical under the action of a force perpendicular to the string. Find the magnitude of each of the force and the tension in the string.

Equilibrium of a body under the effect a set of coplanar forces meeting at a point

Activity

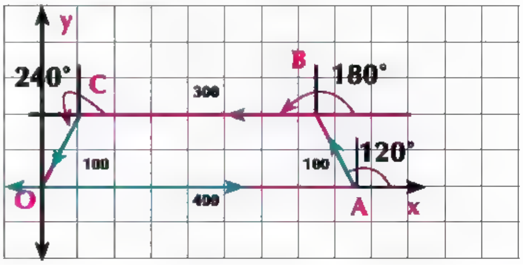
Polygon of forces

Using the (Geo Gebra) program:

Represent the forces of magnitudes 400 , 100 , 300 , 100 dyne which act with polar angles of measures: 0° , 120° , 180° , 240° respectively What do you notice?

We notice that:

➢ The terminal point of the last force is coinciding on the initial point of the first force in the polygon of forces as shown in the figure, This could be the closed polygon of forces OABC.



We conclude from this activity that:

The necessary and sufficient condition of equilibrium of a set of coplanar forces meeting at a point is that these forces are to be represented by a closed polygon taken in the same cyclic order.

The analytical method in the study of the equilibrium of a set of coplanar forces meeting at a point.

In the previous activity it is possible to find the two components of each of these forces in the directions of the x-axis and the y-axis as follow:

$x = 400 \cos 0° + 100 \cos 120° + 300 \cos 180° + 100 \cos 240°$

$= 400 - 100 \times \frac{1}{2} - 300 - 100 \times \frac{1}{2} = \text{zero}$

$y = 400 \sin 0° + 100 \sin 120° + 300 \sin 180° + 100 \sin 240°$

$= 0 + 50\sqrt{3} + 0 - 50\sqrt{3} = \text{zero}$

We conclude that in order that a set of coplanar forces meeting at a point to be in equilibrium, the following conditions must be satisfied:

- The sum of the algebraic components in the direction $\overrightarrow{OX}$ = zero
- The sum of the algebraic components in the direction $\overrightarrow{OY}$ = zero

then $x = 0 ,\ y = 0$

we can define the necessary condition of equilibrium of a set of coplanar forces meeting at a point as the following: If a body is in equilibrium under the effect of a set of coplanar forces meeting at a point ,then the algebraic sum of the algebraic components of the forces in two perpendicular directions equals zero

Example

8 If $\overrightarrow{F_1} = 5\overrightarrow{i} - 3\overrightarrow{j}$, $\overrightarrow{F_2} = -7\overrightarrow{i} + 2\overrightarrow{j}$, $\overrightarrow{F_3} = 2\overrightarrow{i} + \overrightarrow{j}$

Prove that the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ are in equilibrium.

Solution

$\because \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$

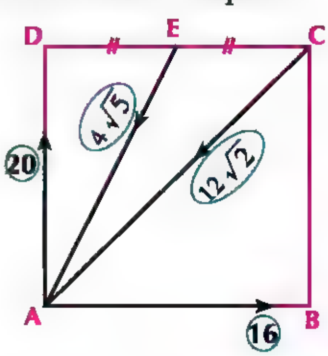
$\therefore \overrightarrow{R} = (5 - 7 + 2)\overrightarrow{i} + (-3 + 2 + 1)\overrightarrow{j} = \overrightarrow{0}$ **,then the set of forces are in equilibrium.**

Try To Solve

7 If the forces $\overrightarrow{F_1} = 4\overrightarrow{i} - 3\overrightarrow{j}$, $\overrightarrow{F_2} = -a\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_3} = -6\overrightarrow{i} + b\overrightarrow{j}$ are meeting at a point and they are in equilibrium, find the values of a , b.

Example

9 **In the opposite figure:** ABCD is a square, The forces of magnitudes: 16 , 20 , $12\sqrt{2}$, $4\sqrt{5}$ newton act, in the directions $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CA}$, $\overrightarrow{EA}$ respectively. where E is the midpoint of $\overline{CD}$. Prove that these forces are in equilibrium.



Solution

From the opposite figure, we notice that the forces 16, 20, $12\sqrt{2}$, $4\sqrt{5}$ have polar angles of measures: 0° , 90° , 225° , (180° + θ)

$\therefore x = 16 \cos 0° + 20 \cos 90° + 12\sqrt{2} \cos 225° + 4\sqrt{5} \cos (180° + \theta)$

$x = 16 + 0 - 12\sqrt{2} \times \frac{1}{\sqrt{2}} - 4\sqrt{5} \times \cos\theta$

$x = 16 - 12 - 4\sqrt{5} \times \frac{1}{\sqrt{5}} = 0$

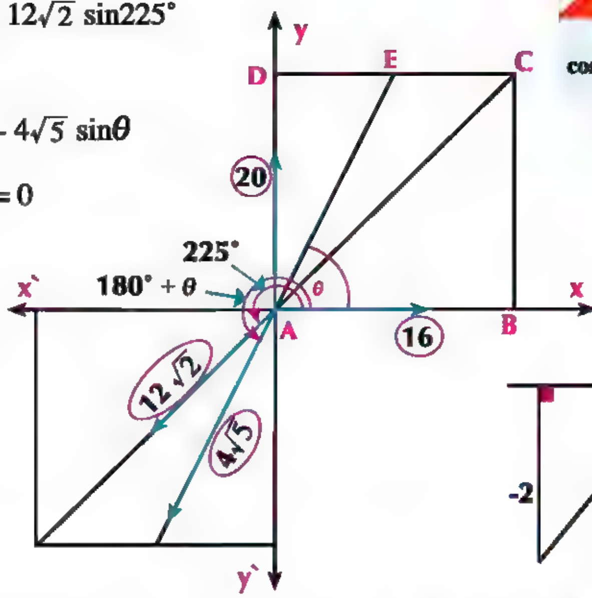
$y = 16\sin 0^\circ + 20\sin 90^\circ + 12\sqrt{2}\sin 225^\circ + 4\sqrt{5}\sin(180^\circ + \theta)$

$y = 0 + 20 - 12\sqrt{2} \times \frac{1}{\sqrt{2}} - 4\sqrt{5}\sin\theta$

$y = 20 - 12 - 4\sqrt{5} \times \frac{2}{\sqrt{5}} = 0$

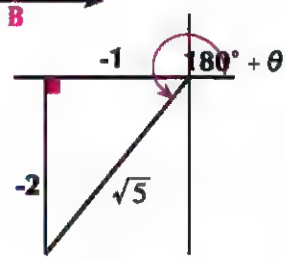
$\therefore x = 0 \quad , \quad y = 0$

$\therefore$ The set of forces are in equilibrium.



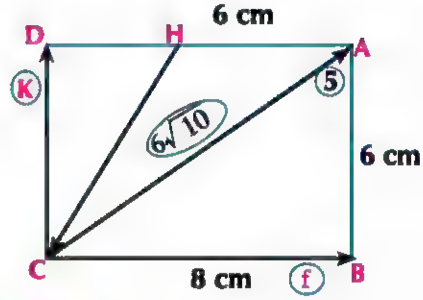
Note that

$\cos(180^\circ + \theta) = -\cos\theta$



Try To Solve

8) **In the opposite figure:** The forces of magnitudes F, 5, K, and $6\sqrt{10}$ N are in equilibrium and they act in the rectangle ABCD in the directions $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{HC}$ Such that: AB = 6cm, BC = 8cm, AH = 6 cm. Find the values of F, K.



Exercises (1 - 4)

Complete the following:

1) The necessary and sufficient condition for equilibrium of a set of coplanar forces meeting at a point is to be represented geometrically by ....

2) The condition for equilibrium of a set of coplanar forces, meeting at a point is to be .... , ...

3) If $\overrightarrow{F_1} = 4\overrightarrow{i} + b\overrightarrow{j}$, $\overrightarrow{F_2} = -7\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_3} = a\overrightarrow{i} - 3\overrightarrow{j}$ are in equilibrium, so: a = ... , b = ...

4) If the force of magnitude F is in equilibrium with two perpendicular forces of magnitude 3, 4 newton so, the magnitude of F = ...

5) If three coplanar and equilibrium forces are completely represented by the sides of triangle taken in one cyclic order, then the lengths of the sides of the triangle are proportional with .. ..

6 Each figure from the following figures represents a set of coplanar equilibrium force meeting at a point. Find the value of the unknown either it is a force or a measure of angle .

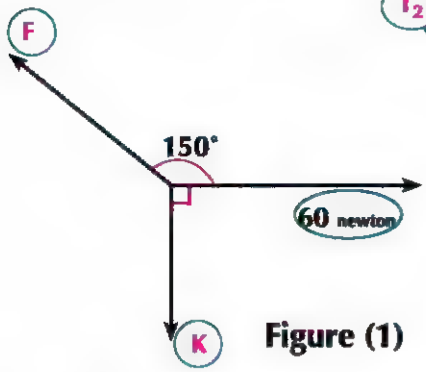


Figure (1)

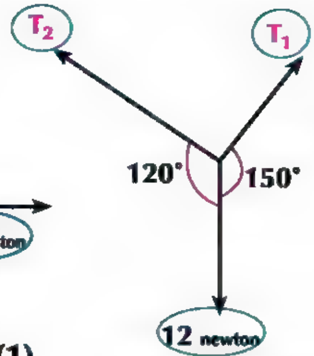


Figure (2)

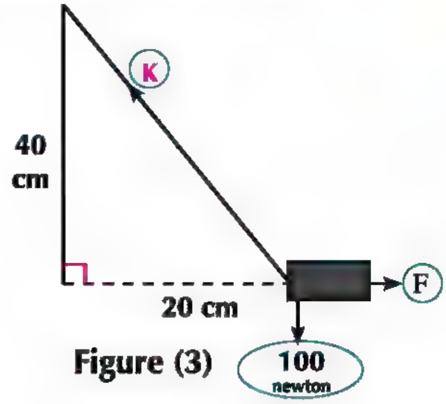


Figure (3)

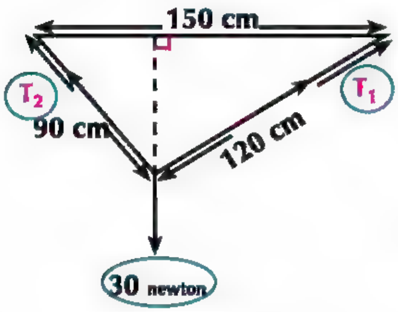


Figure (4)

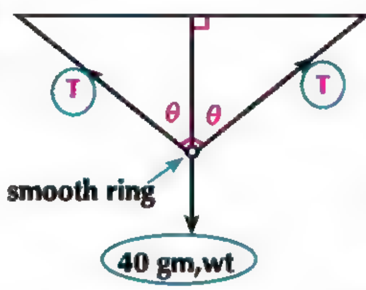


Figure (5)

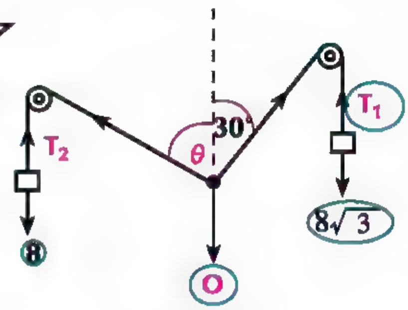


Figure (6)

7 AB is a uniform lader with weight 12 kg.wt rests with its upper end A on a smooth vertical wall and with its lower end B on a rought horizontal ground such that its upper end apart 4m from the ground and its lower end apart 3m from the vertical wall. Find in the case of equilibrium the presure on the wall and the ground

8 AB is uniform rod with length 60cm and weight 40 Newton conected to a hang on the vertical wall at A. If the rod keept in equilibrium by a light string connected to the rod at B and with point C on the wall just above A and at adistance 60 cm from A. Find the tension on the string and the reaction on the hang at A.

9 A homogeneous sphere rests on two parallel rods lie on the same horizontal plane and the distance between them equals the radius of the sphere. Find the pressure on the two rods if the weight of the sphere equals 60 Newton.

10 AB is a uniform rod whose weight is W kg. wt attached by its end A to a hang fixed on a vertical wall. If a horizontal force $\overrightarrow{F}$ acts on the rod at B and the rod get equilibrium when it inclined to the vertical by an angle of measure 60. Find the magnitude of $\overrightarrow{F}$ and the reaction of the hang.

(11) A weight of magnitude 60 gm.wt is suspended from one end of a string of length 28 cm. The other end is fixed at a point in the ceiling of the room, A force acts on the body so that the body became in equilibrium when it is about 14 cm vertically down the ceiling, If the force is in equilibrium position when it is normal to the string . Find the magnitude of each of the force and the tension in the string.

(12) A weight of magnitude 200gm.wt is hanged (suspended) by two strings of lengths 60 cm, 80cm from two points on one horizontal line. The distance between them is 100 cm. Find the magnitude of the tension in each of the two strings.

(13) A particle of weight 200 gm.wt is hanged (suspended) by two light strings. One of the them inclines to the vertical with an angle of measure θ and the other string inclines to the vertical with an angle of measure 30°. If the magnitude of the tension in the first string equals 100 gm.wt, find θ and magnitude of the tension of the second string.

(14) A body of weight 800 gm.wt is placed on a smooth plane inclined to the horizontal by angle of measure θ so that $\sin\theta = 0.6$. The body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

(15) A body of weight (W) newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and the body is kept in equilibrium by the effect of force of magnitude 36 newton acts in direction of the line of the greatest slope upwards. Find the magnitude of the weight of the body and the magnitude of the reaction of the plane.

(16) A smooth metal sphere of weight 3 Newton at rest (stable) between a smooth vertical wall and a smooth plane inclined to the vertical wall with angle of measure 30°. Find the pressure on each of the vertical wall and the inclined plane.

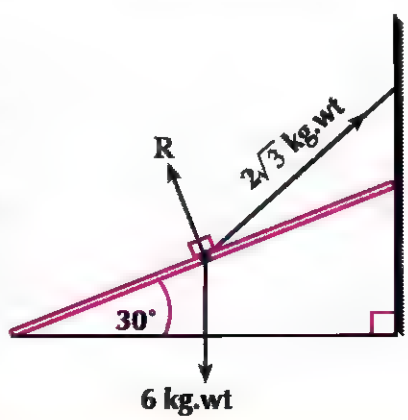
(17) A rod of length 50 cm and weight 20 newton was hanged (suspended) from its terminals with two strings such that the two ends are fixed in one point. If the length of the two strings are 30 cm , 40 cm respectively. Find the tension in each of the two strings.

(18) Five forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K kg.wt are in equilibrium and act on a particle in the directions of the east, the north, the western north, the western south and the south respectively. Find the magnitudes of F , K.

(19) Coplanar forces of magnitudes 5 , 4 , F , 3 , K , 7 kg.wt act on a particle and the angle between every two consecutive forces from them is 60°. Find the magnitude of each of F , K that make the set in a state of equilibrium.

Creative thinking:

(20) **In the opposite figure** A body of weight 6 kg.wt is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and kept in equilibrium by a tension force (T) of magnitude $2\sqrt{3}$ kg.wt the tension force acts in a string one of its ends fixed to the body and the other in a vertical wall. Find the measure of the angle between the string and the plane and the magnitude of the reaction of the plane on the body.



Unit Summary

Fundamental quantities and its measurements units (SI)

Basic quantities	length	Mass	Time
Basic units	meter (m)	kilogram (kg)	second (s)

Derived quantities :

Unit	velocity (v)	Acceleration (a)	Force (F)
Relation with basic units	$V = \frac{s}{t}$	$a = \frac{v}{t}$	$F = m \times a$
Measurement	m/sec	m/sec^2	Newton

Some transforming of derived quantities:

- 1km/h= $\frac{5}{18}$ m/sec , 1 km/h = $\frac{250}{9}$ cm /sec , 1 m/sec = $\frac{18}{5}$ km/h , 1 cm/sec = $\frac{9}{250}$ km/h
- Newton = 10^5 Dyne , Dyne = 10^{-5} Newton , 1 kg.wt = 9.8 Newton, 1 gm.wt = 980 Dyne.

Statics: The branch of mechanics that deals with bodies at rest or forces in equilibrium.

Rigid bodies : A body whose deformation is neglected whatever the action of the external effect.

Force: The effect of a natural body upon another one.

Properties of a force: The effect of a force is determined by:

1- Magnitude. **2-** Direction. **3-** Point of action. (line of action)

- If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces , the angle between their lines of actions α, their resultant $\overrightarrow{R}$ which inclined by an angle of measure θ with the direction of $\overrightarrow{F_1}$ then:
- $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$ $\qquad \tan\theta = \dfrac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha}$

Or by using sine rule: $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{R}{\sin\alpha}$

- The maximum magnitude of resultant of two forces F_1 , $F_2 = F_1 + F_2$ acts in their directions.
- The minimum magnitude of resultant of two forces F_1 , $F_2 = |F_1 - F_2|$ acts in the direction of the big force.
- If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two components of a force $\overrightarrow{R}$, they make with the direction of $\overrightarrow{R}$ two angles of measures θ_1, θ_2 respectively ,then:

$$\frac{F_1}{\sin\theta_2} = \frac{F_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$$

- If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two perpendicular components of a force $\overrightarrow{R}$, If $\overrightarrow{R}$ inclines by an angle of measure θ with the direction of $\overrightarrow{F_1}$ then $F_1 = R\cos\theta$, $F_2 = R\sin\theta$

1 - 4 Unit Summary

- **Polygon of forces:** If a set of coplanar forces meeting at a point are represented by sides of a polygon taken in the same cyclic order , then the magnitude of the resultant of these forces equals the length of side that closed this polygon in the opposite cyclic order.

- In a perpendicular coordinate plane if a set of coplanar forces that meeting at a point act at this point, and the algebraic sum of the components of these forces in the two perpendicular direction are x ,y , then: $R = \sqrt{x^2 + y^2}$, $\tan \theta = \frac{y}{x}$ where θ is the angle between the resultant and x component.

- If a set of coplanar forces meeting at a point are completely represented by a sides of a closed polygon , taken in the same cyclic order , then the forces are equilibrium.

- A set of coplanar forces meeting at a point is to be in equilibrium, if

  1) The sum of the algebraic components in the direction $\overrightarrow{OX}$ = zero
  2) The sum of the algebraic components in the direction $\overrightarrow{OY}$ = zero.

- A body is in equilibrium under the effect of only two forces, means : The two forces are equal in magnitude, opposite in direction and their line of action are on the same straight line.

- **Transfer the point of action of the force:** if a force acts on a rigid body ,then its possible to transfer its point of action to another point in the body on the same line of action of the force without any change of its effect on the body.

- **The equilibrium of a body under the effect of three forces:** If a set of coplanar forces meeting at a point are represented by the sides of triangle taken in the same cyclic order, then these forces are equilibrium

- **Triangle of forces rule:** If three forces acting at a point are in equilibrium and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the sane cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

- **Lami's rule:** If three forces meeting at a point and acting up on a particle are in equilibrium, then the magnitude of each forces is proportional to the sine of the angle bevtween the two other forces.

- If a set of coplanar forces are completely represented by the sides of a closed polygon of forces taken in the same cyclic order, then these forces are equilibrium

General Exercises

For more exercises please visit the website of the Ministry of Education.

Accumulative test

Short answers questions:

1. Complete the following:

   a The scalar quantity to be completely define we need to know ....

   b The vector quantity to be completely define we need to know ....,..................

   c The directed line segment is the line segment with..................,..................,..................

   d Two directed line segments are equivalent if they have ....

   e The polar form to the vector $\overrightarrow{M} = 3\overrightarrow{i} + 4\overrightarrow{j}$ is ....

   f The vector that represents the force with magnitude 20 kg.wt in the direction 30° south of east is written as ....

2. In the opposite figure: ABCD is a parallelogram whose diagonals intersected at M:

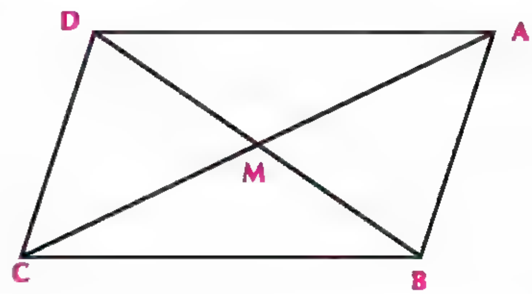
   a) $\overrightarrow{AB} + \overrightarrow{BC} =$ ....

   b) $\overrightarrow{DA} + \overrightarrow{DC} =$ ....

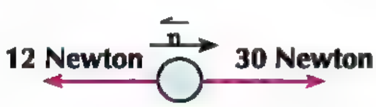
   c) $\overrightarrow{AM} + \overrightarrow{CM} =$ ....

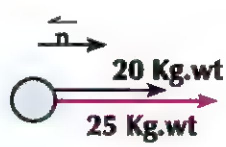
   d) $\overrightarrow{AB} + 2\overrightarrow{BM} =$ ....

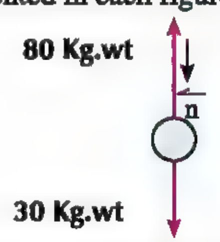
   e) $\overrightarrow{AB} - \overrightarrow{AM} =$ ....



3. Write in term of the unit vector $\overrightarrow{n}$ the resultant of the forces represented in each figure:







4. In each of the following forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ act at a particle ,determine the direction and the magnitude of the resultant of each of them:

   a F_1= 15 Newton acts in the eastern direction, F_2 = 40 Newton acts in the western direction.

   b F_1 = 34 gm.wt acts in direction of northern east , F_2 = 34 gm.wt acts in direction of southern west.

   c F_1 =50 Dyne acts in direction of western north F_2 = 50 Dyne acts in direction 30° of the southern east .

   d F_1 = 30 Newton acts in direction 20° of the eastern north , F_1 = 30 Newton acts in direction 70° of northern east .

5. $\overrightarrow{F_1} = 7\overrightarrow{i} - 5\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} + 3\overrightarrow{j}$, $\overrightarrow{F_3} = -4\overrightarrow{i} + (b-3)\overrightarrow{j}$ act at a particle . Find the values of a and b if:
   a. the resultant of the forces equals $4\overrightarrow{i} - 7\overrightarrow{j}$
   b. the forces are equilibrium

Long answers questions

6. Two forces of magnitude $8\sqrt{3}$, 8 newton act at a particle and enclose between them an angle of measure 150°. Find the magnitude of their resultant and the measure of the angle which it makes with the first force.

7. Two forces of magnitude 30 , 16 newton act at a particle, if the magnitude of their resultant is , 26 newton. Find measure of the angle between these two forces.

8. Two forces of magnitude 2 , F newton and the measure of the angle between them is 120° Find F when:
   A. Magnitude of the resultant equals F.
   B. The direction of the resultant is perpendicular to the second force.
   C. The resultant bisects the angle between the two forces.

9. Resolve a force of magnitude 60 newton into two forces of equal magnitude and the measure of the angle between their lines of action is 60°.

10. Find the magnitude of the two perpendicular components, to a weight of body placed on horizontal plane if its magnitude = 80 newton if it is known that one of them inclined to the horizontal with angle of measure 30° downwards.

11. Three forces of magnitudes 2F, 4F , 6F Newton act at a particle in directions parallel to the sides of an equilateral triangle and in the same cyclic order. Find the direction and the magnitude of the resultant.

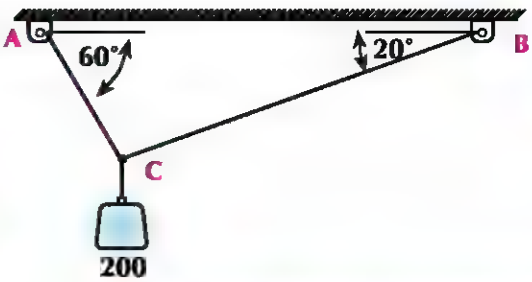
12. ABCD is a rectangle in which AB = 8cm , BC = 6 cm , E $\in \overline{CD}$ in which ED = 6cm forces of magnitude 6, 20, $13\sqrt{3}$, 2 newton act in $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AE}$,$\overrightarrow{AD}$ respectively. Find the magnitude and direction of the resultant of these forces.

13. A weight of magnitude 80 gm.wt is suspended by a string fixed in a vertical wall. if the weight is pulled by a force perpendicular to the string till it becomes in a position inclined on the wall by an angle of measure 30°. Find the magnitude of the force and the tension in the string in the state of equilibrium.

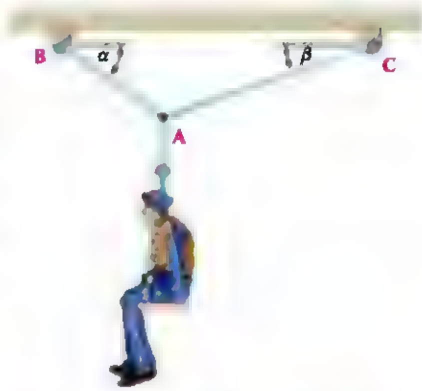
14. A weight of magnitude 20 kg.wt is placed on a smooth plane inclined to the horizontal by an angle θ, where $\cos\theta = \frac{4}{5}$ and it was prevented from sliding under the effect of a horizontal force of magnitude (F). Find F and the reaction of the plane.

15. A uniform rod rests with its ends on two smooth planes inclined to the horizontal by two angles of measure 60°, 30°. Find the measure of the angle that the rod makes with the horizontal in the equilibrium state, If the magnitude of weight of the rod equals 24 newton, Find the magnitude of the reaction for each of the two planes.

(16) The opposite figure represents a weight of magnitude 200 Newton hanged vertically at a point C by two strings $\overline{BC}$ and $\overline{AC}$ which make with the horizontal the angles of measures 20°,60° respectively find in the state of equilibrium the tension in the two strings to the nearest newton



(17) **Join with navigation :** The operation of saving a nautilus is done by using the captain chair which is hanged in a bully .Two ropes $\overline{AB}$ and $\overline{AC}$ are passing over the bully making two angles α, B with the horizontal whose measures are 25°, 15° respectively. If the tension in the rope $\overline{AB}$ equals 80 Newton ,find the weight of the nautilus and the chair together and the tension in the rope $\overline{AC}$ in the state of equilibrium.



If you can not answer these questions you can use the following table:

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	If you can not answer question number
Lesson (4) 2nd sec	Lesson (4) 2nd sec	Lesson (4) 2nd sec	Lesson (4) 2nd sec	Lesson (4) 2st sec	Lesson (3) 2nd sec	Lesson (3) 2nd sec	Lesson (2) 2nd sec	Lesson (2) 2nd sec	Lesson (1) 2nd sec	Lesson (1) 2nd sec	Lesson (1) 2nd sec	vectors 1st sec	vectors 1st sec	vectors 1st sec	vectors 1st sec	vectors 1st sec	go back to

Unit Two

Dynamics

introduction

Dynamics concerns with the study of the object movements and forces causing that movement. Dynamics is divided into kinematics and kinetics in this unit, we are going to study kinematics. Kinematics describes the object movements disregarding the forces cting on . kinematics has its own applied importance in the practical life such as accounting solar and lunar aclipses before they occur. Furthermore, it helps to direct the missles to their targets very accurately and to identify the pathway of spaceships or sattclites and their landing ground points. Kinematics is also used to design mechanical machinaries. As a resut, we are going to study the object movement and the phenomena associated with this movement and its causes.

Unit objectives

By the end of the unit the students should be able to:

- Recognize the concept of the partice as a geometrical point.
- Understand the meaning of the translational motion of a particle from a position to another.
- Realize that the tmaslational motion is taken place if all points of the body move in parallel during the motion .
- Distinguish between distance and displacement .
- Recognize the concept of the uniform velocity (velocity vector - unifotm motion - Average velocity vector - instantaneous velocity vector - measuring units of velocity.
- Distinguish between the concepts of the average velocity vector and the magnitude of the average velocity in the rectilinear motion in a fixed direction.
- Apply the concepts of the velocity, average velocity, acceleration in modelling physical and life application including the movements of (planes, nechets, satellites) as activities .
- Recognize the concept of the relative velocity.
- Deduce the laws of motion with uniform acceleration if a body moves with uniform acceleration and : $v = u + at$, $s = ut + \frac{1}{2} at^2$, $v^2 = u^2 + 2as$
- Recognize the vertical motion under gravity.
- Recognize the applications on the laws of the straight motion with uniform acceleration
- Recognize the laws of the vertical motion"up and down under gravity.
- Recognize the gravity (Universal gravitation law).
- Universal Gravitational constant.
- Recognize the graphical representation of the (displacement - time curve), (velocity - time curve).
- Use graphical calculator to represent the relation between (displacement - time) and (velocity - time) as an activity

Key Terms

- Rectilinear Motion
- Distance
- Vector Velocity
- Average Velocity
- Average Speed
- Relative Velocity
- Vertical Motion
- Universal Gravitation
- Displacement
- Uniform Velocity
- Instantaneous Velocity
- Position Vector
- Uniform Acceleration
- Free fall
- Gravity

Materials

- Scientific calculator
- Graphical calculator
- Graphical programs

Lessons of the unit

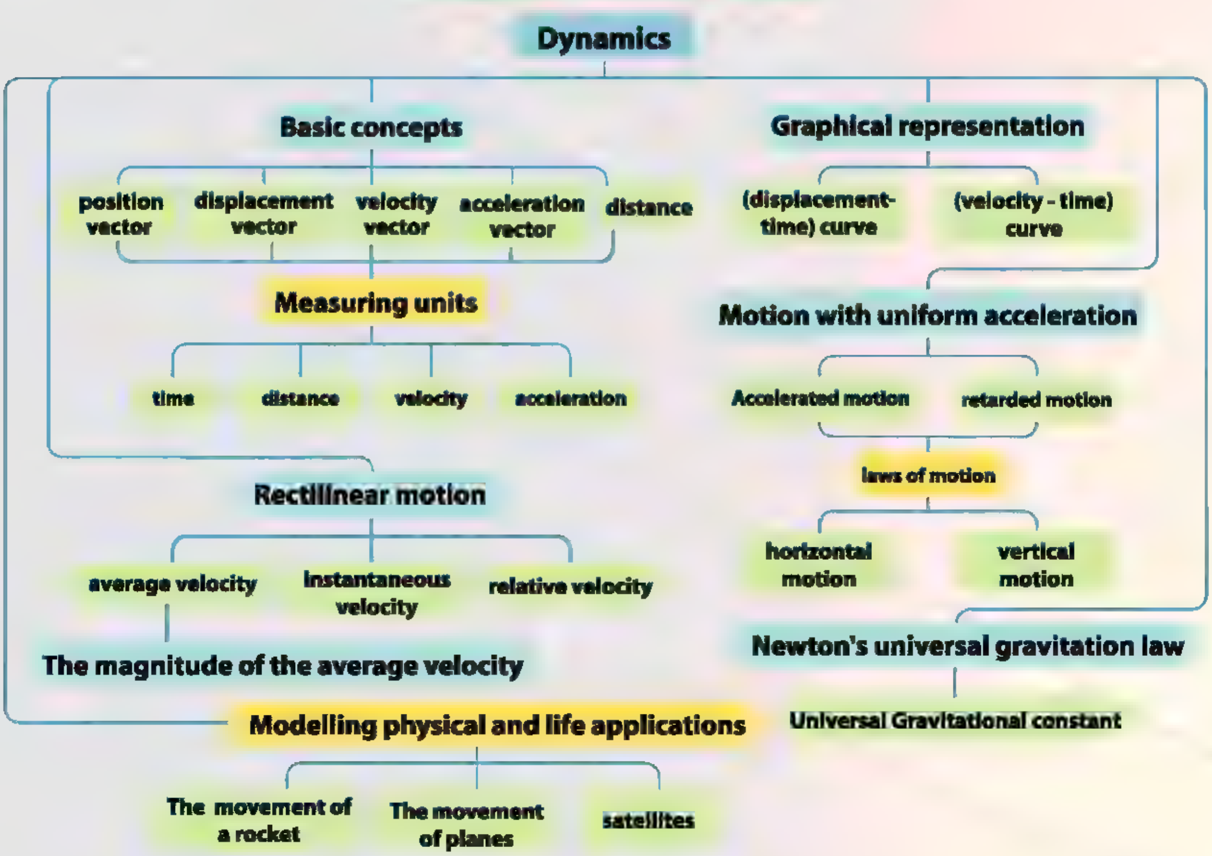
Lesson (2 - 1): Rectilinear motion.

Lesson (2 - 2): Uniformly accelerated rectilinear motion.

Lesson (2 - 3): Vertical motion under the effect of gravity (Free fall).

Lesson (2 - 4): Newton's universal gravitation law.

Chart of the unit



2 - 1 Rectilinear motion

- The relation between position vector and displacement vector
- Average velocity
- Instantaneous velocity
- The relative velocity

Key terms

- Rectilinear Motion
- IS
- Displacement Vector
- Position Vector
- Velocity Vector
- Uniform motion
- Average Velocity
- Instantaneous Velocity
- Relative Velocity

Tools

- Squared paper
- Scientific calculator
- Drawing programs

Introduction:

You have learned some measurement systems that began in people's lives. Until the decimal system has been adopted which was invented by the French in 1970 and continued until the consolidated international system IS. It is derived from the word international system of units. This system is formed of basic units in mechanics (mass , length , time) as well as from the derived units which come from products of powers of basic units according to some algebraic relations as (velocity , acceleration , force).

Motion

Rest and Motion:

If a body changes its position with respect to another body related to time, then it said that the first body is in a movement state with respect to the other body. If the relative positions of the two bodies do not change over time, then they will be at rest to each other.

Rest and Motion are two relative concepts. We know that trees and houses are at rest but they seem to be in motion with respect to a moving train.

Motion and its Types

There are many types of motion as the translational motion, rotational motion and harmonic motion .For example, the projected football translates from a position to another position. It may rotate around itself, so it moves in a translational motion and a rotational motion at the same time. The falling water drops move in a translational motion and a harmonic motion at the same time.

We will study only the translational motion assuming the motion of a very small body called particle ,the particle is considered a geometrical point without any dimensions to avoid the theoretical complexes resulted in the rotational and harmonic motions which will be listed in this study.

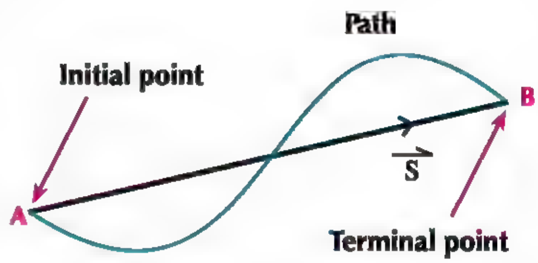
Translational Motion

In the translational motion the body moves between two points, the first point is called the initial point and the second point is called the terminal point. The motion of the body in a straight line is an example.

Distance

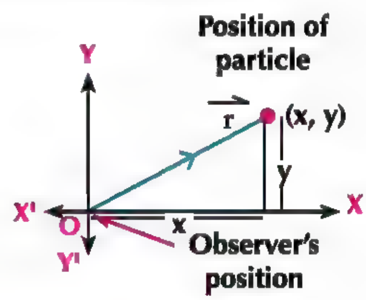
If a train moves from Cairo to Mansoura, then it covers a distance of 126 Km. The distance is a scalar quantity. It is sufficient to determine the magnitude of the distance. If the magnitude of the distance between the two cities is 126 km, then the number 126 represents a numerical value and (km) is the measuring unit of distance.

Displacement vector



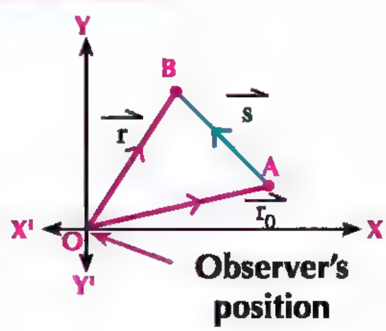
The displacement vector is the vector represented by the directed line segment $\overrightarrow{AB}$ where its starting point A coincides with the initial position of the body and its terminal point B on its end. The displacement vector $\overrightarrow{AB}$ is denoted by $\overrightarrow{S}$, the norm of the displacement vector $\overrightarrow{AB}$ is denoted by $\| \overrightarrow{AB} \|$ and that may not be equal to the distance covered by the body during its motion.

Position vector



The position vector of a praticle is the vector whose initial point coincides with the position of the observer (o) and its end point coincides with the position of the particle. The position vector of the particle is denoted by $\overrightarrow{r}$ where $\overrightarrow{r} = x \overrightarrow{i} + y \overrightarrow{j}$ $\overrightarrow{i}$, $\overrightarrow{j}$ are two perpendicular fundamental unit vectors.

Relation between position vector and displacement vector:



If (O) is the position of the observer and $A(x_1, y_1)$, $B(x_2, y_2)$ are the two positions of the particle at two successive moments, then $\overrightarrow{AB}$ is the displacement vector of the particle let it $\overrightarrow{S}$. If we denote the position vector at the moment t with the symbol $\overrightarrow{r_0}$, and the position vector the moment (t + E) with the symbol $\overrightarrow{r}$, then $\boxed{\overrightarrow{S} = \overrightarrow{r} - \overrightarrow{r_0}}$

$$\overrightarrow{S} = (x_2 \overrightarrow{i} + y_2 \overrightarrow{j}) - (x_1 \overrightarrow{i} + y_1 \overrightarrow{j})$$

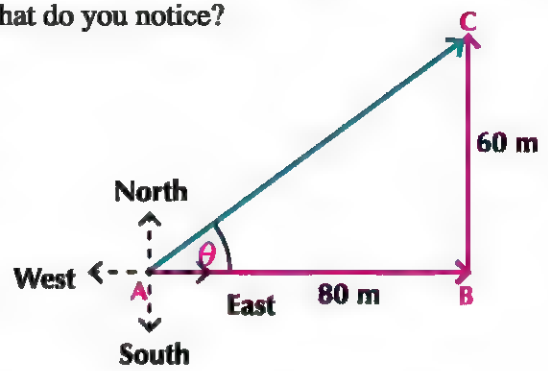
$$= (x_2 - x_1) \overrightarrow{i} + (y_2 - y_1) \overrightarrow{j} \quad , \quad \| \overrightarrow{S} \| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\therefore$ $\overrightarrow{S} = \| \overrightarrow{S} \| \ \overrightarrow{n}$, $\overrightarrow{n}$ is the unit vector in direction of $\overrightarrow{S}$ (direction of motion)

Example

(1) A runner moves 80 m. due to East, then he moves 60 m. due to North. Find the distance and the displacement covered by the runner. What do you notice?

Solution



The total distance covered by the runner is the sum of the two distances from A to B then from B to C.

The distance = A B + B C = 80 + 60 = 140 m

The displacement is represented by the directed line segment $\overrightarrow{AC}$

From Pythagoras :

$$AC = \sqrt{(80)^2 + (60)^2} = \sqrt{10000} = 100, \quad \tan\theta = \frac{60}{80}, \text{ then } \theta = 36^\circ\, 52'\, 12''$$

So that: the magnitude of the displacement = 100 m and it works in the direction of $36^\circ\, 52'\, 12''$ north of the east.

We notice that:

- The covered distance is a scalar quantity (identified in terms of its value), while the displacement is a vector quantity (identified in terms of its magnitude (value) and its direction).
- The magnitude of the displacement vector $\leqslant$ the covered distance.

Try to solve

(1) A bicyclist moves 6 Km. due to West, then he moves 8 Km. with angle 60°south of the west. Find the distance and the displacement covered by the bicyclist.

(2) Critical thinking: An ant ascends vertically a wall with height 3m, then it return back to its start point. Find the distance and the displacement covered by the ant.

Example

(2) A particle moves so that its position vector $\overrightarrow{r}$ is given as a function in time in terms of the fundamental unit vectors $\overrightarrow{i}$, $\overrightarrow{j}$ with the relation: $\overrightarrow{r}(t) = (3t + 2)\overrightarrow{i} + (4t - 1)\overrightarrow{j}$.

Find the magnitude of the displacement vector till the moment t = 4

Solution

$$\overrightarrow{r}(0) = 2\overrightarrow{i} - \overrightarrow{j}, \quad \overrightarrow{r}(4) = (3 \times 4 + 2)\overrightarrow{i} + (4 \times 4 - 1)\overrightarrow{j} = 14\overrightarrow{i} + 15\overrightarrow{j}$$

$$\therefore \overrightarrow{S} = \overrightarrow{r}(4) - \overrightarrow{r}(0)$$

$$= (14 - 2)\overleftarrow{i} + (15 + 1)\overrightarrow{j} = 12\overrightarrow{i} + 16\overrightarrow{j}$$

$$, \|\overrightarrow{S}\| = \sqrt{144 + 256} \qquad , S = 20 \text{ length units}$$

Try to solve

(3) In the previous example : Find the magnitude of the displacement vector from $t = 1$ to $t = 3$.

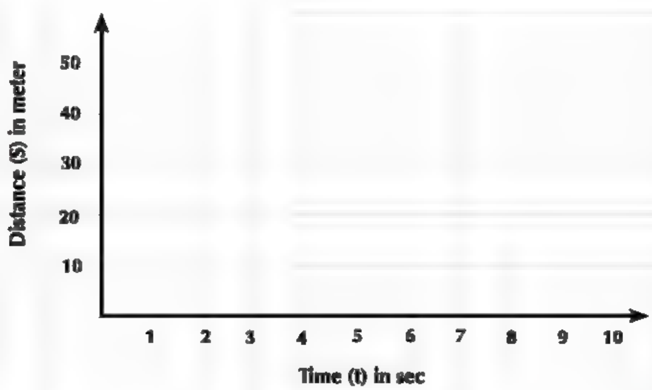
Activity

The curve of (Distance – Time)

The following table shows the relation between the time in seconds and the distance in meter for a runner.

Time in seconds	0	2	4	6	8	10
Distance in meters	0	10	20	30	40	50

1. In the graph paper, determine the time on x- axis and the distance on y-axis.
2. Represent graphically the coordinates of the points in the table.
3. Use the ruler to draw the best straight line that passes through the majority of the points in the graph.
4. Use the line that represents the relation between the distance and the time in the times shown in the table. Can you determine the following:
   - a) The distance covered by the runner after 3 sec ?
   - b) The time taken by the runner to cover 45 meter ?
5. Can you determine the slope of the straight line that represents the type of the motion of the runner? Explain that.



Velocity

If two runners compete in a certain time interval, then the runner who covers a longer distance is faster than the runner who covers a shorter distance. Velocity can be measured by the covered distance during a certain time interval without determining its direction.The Speedometer in front of the driver determines the magnitude of the velocity without determining the direction of the car.

Remember that

$1\text{km/hr} = \frac{5}{18}\text{ m/sec}$

$1\text{m/sec} = \frac{18}{5}\text{ km/hr}$

Try to solve

4. a. Convert 90 km/hr into m / sec b. Convert 15m/sec into km/hr

5. Complete the following table:

18km/hr	54km/hr	....km/hr	90km/hr	...km/hr	180km/hr
5 m/sec	...m/sec	20m/sec	...m/sec	30m/sec	...m/sec

$\times \frac{5}{18}$ (km/hr → m/sec), $\times \frac{18}{5}$ (m/sec → km/hr)

Velocity vector

The velocity vector of a particle is the vector whose magnitude equals the value of the velocity, and its direction is the same as the direction of motion.

Verbal Expression:

1- Compare between the velocity and the velocity vector in:
  a. Definition. b. The type of the quantity (scalar – vector)

Uniform motion and variable motion

Uniform motion: is the case in which the magnintude and the direction of the velocity vector is constant.

So, we will list two important remarks about the uniform motion.

1 - The fixity of the direction of velocity vector: means that the body moves in a fixed direction.

2 - The fixity of the magnitude of the velocity vector: means that the body covered equal distances in equal time intervals in the direction of the motion.

Variable motion: If the motion is not uniform, then the motion is called a variable motion.

In the variable motion the velocity vector of the body changes its magnitude or its direction or both of them from an instant to another.

average velocity

If a car made a trip from Cairo to Hurgada, then the distance between the two towns according to the path of the car reaches 510 km. If the car moves with variable velocities between the two towns and the total time for this trip is 6 hours , So to calculate the average velocity of the car during this trip we will find that:

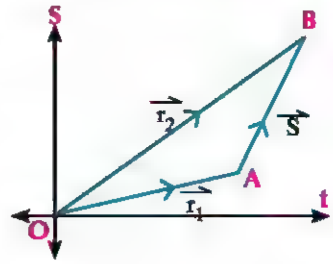
$$\text{The average velocity } V_A = \frac{\text{Total distance}}{\text{Total time}} = \frac{510}{6} = 85 \text{ Km / hr}$$

So that :

The average velocity is determined by the total covered distance during the whole trip divided by the total time taken in this trip.

Vector of the average velocity

If a particle takes two positions A and B between two successive moments t_1 and t_2 respectively and let $\overrightarrow{S}$ be the displacement vector during the time interval $(t_2 - t_1)$, $\overrightarrow{V_A}$ is known as the vector of the average velocity for that particle through this time interval and it is:



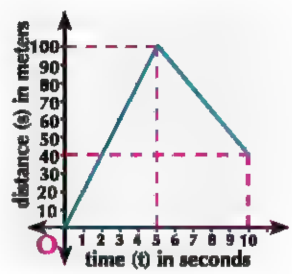
$$\overrightarrow{V_A} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1} = \frac{\overrightarrow{s}}{t_2 - t_1}$$

Example

(3) The opposite figure shows the relation between time and distance for the motion of a cyclist the motion starts from the point (0) in a straight line , find:

a The vector of the average velocity.

b The average velocity.



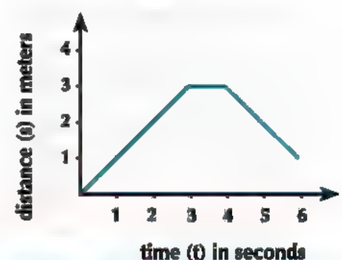
Solution

a $\overrightarrow{V_A} = \frac{40\ \overrightarrow{n}}{10} = 4\ \overrightarrow{n}$ and its magnitude is 4 m/sec.

b $V_A = \frac{100 + 60}{10} = 16$ m/sec.

Try to solve

(6) The opposite figure shows a chart for the curve (distance - time) of a rat escapes from a cat. Redraw this figure if the rat escapes from the cat with double its speed.

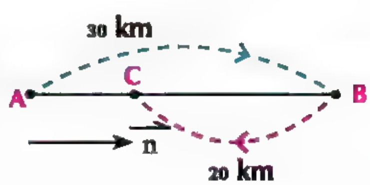


Example — Calculating the average velocity and the average velocity vector

(4) A cyclist covered 30 km on a straight road with velocity 18 km/hr., and then he returned on the same road and covered 20 km in the opposite direction with velocity 15 km/hr. Find the average velocity and the average velocity vector during the whole journey.

Solution

If the cyclist starts his motion from position A towards position B in the first stage, then he returned from B to C in the second stage, let $\overrightarrow{n}$ be the unit vector in the direction of $\overrightarrow{AB}$.



Time of the first stage $t_1 = \frac{d}{v_1}$ then : $t_1 = \frac{30}{18} = \frac{5}{3}$ hours.

Time of the second stage $t_2 = \frac{20}{15} = \frac{4}{3}$ hours.

The total time for the whole journey $= \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3$ hours

The displacement $\overrightarrow{S} = 30\ \overrightarrow{n} - 20\ \overrightarrow{n} = 10\ \overrightarrow{n}$

$\because \overrightarrow{V_A} = \dfrac{\overrightarrow{S}}{t}$ $\therefore \overrightarrow{V_A} = \dfrac{10\ \overrightarrow{n}}{3} = 3\frac{1}{3}\ \overrightarrow{n}$

$\therefore$ The average velocity vector has the same direction as $\overrightarrow{n}$ as in the same direction of $\overrightarrow{AB}$ and it is magnitude equals $3\frac{1}{3}$ km / hr.

Average velocity $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{30+20}{3} = \dfrac{50}{3}$ km/h

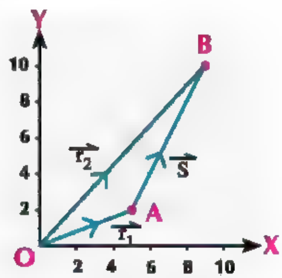
Try to solve

(7) A bicyclist covered a distance 25 km on a straight road with velocity 15 km/hr, then he covered a distance of 7 km in the same direction with velocity 7 km / hr. Find the average velocity vector and the average velocity during the whole journey.

Example

(5) If a particle takes two positions A(5,2) and B(9, 10) between two successive moments 3sec. and 7sec. respectively. Find the direction of the average velocity of the particle during this time interval, then find the magnitude and the direction of this average velocity.

Solution



The opposite figure shows:

Primary position vector $\overrightarrow{OA}$ ($\overrightarrow{r_1}$),

Final position vector $\overrightarrow{OB}$ ($\overrightarrow{r_2}$),

Displacement vector $\overrightarrow{AB}$ ($\overrightarrow{S}$)

where: $\overrightarrow{S} = \overrightarrow{r_2} - \overrightarrow{r_1}$

$\overrightarrow{S} = (9,\ 10) - (5,\ 2)$

$\overrightarrow{S} = (4,\ 8)$

$\because \overrightarrow{V_A} = \dfrac{\overrightarrow{S}}{t_2 - t_1}$

$\therefore \overrightarrow{V_A} = \dfrac{1}{(7-3)}(4\ \overrightarrow{i} + 8\ \overrightarrow{j})$

$\overrightarrow{V_A} = \overrightarrow{i} + 2\ \overrightarrow{j}$ (The vector form of the average velocity)

$\| \overrightarrow{V_A} \| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$ unit of velocity

and it makes a polar angle with $\overrightarrow{Ox}$ whose tangent is 2,

the measure of the angle equals : $63^\circ\ 26'\ 6''$.

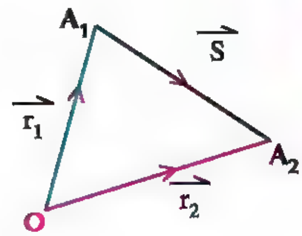
Try to solve

(8) If a particle takes two positions A(7,2) and B(4, 6) between two successive moments 3sec. and 8sec. respectively. Find the direction of the average velocity of the particle during this time interval, and then find the magnitude and the direction of this average velocity.

Instantaneous Velocity

In the opposite figure: $\overrightarrow{V_A} = \dfrac{\overrightarrow{S}}{t_2 - t_1} = \dfrac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1}$.

If the time interval $(t_2 - t_1)$ was very small and the instance (t) is its middle, then the velocity vector in this case is named the instantaneous velocity vector at this instance (t) and is denoted by $\overrightarrow{V}$



Think and discuss

Relative velocity

What do you notice?

- If you sit in a moving train and watch the lamp posts and trees on the side of the road from the window.
- If you sit in a moving car that moves in a certain direction and velocity while you observe other cars moving in the same direction of your car.
- If the other cars are moving in the opposite direction to your car.

We notice from the previous that the movement is a relative concept that differs from an observer to another in another location. In all cases the viewer observes the movements of the other objects conceder himself at rest, even if he is not. The viewer will see the objects moving with velocities that are not the actual velocities, but they are the relative velocities.

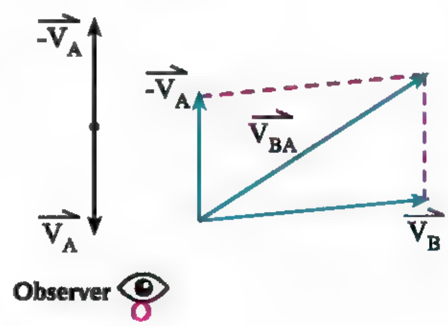
The concept of relative velocity:

The relative velocity for a particle (B) with respect to another particle (A) is the velocity that the particle (B) appears to move with if we assume that the particle (A) is at rest.

Relative velocity vector:

Consider $\overrightarrow{V_A}$, $\overrightarrow{V_B}$ are the two velocity vectors of two bodies A and B with respect to the observer (o) and $\overrightarrow{V_{BA}}$ is the relative velocity vector of B with respect to A.

By adding $(-\overrightarrow{V_A})$ vector to both of the vectors $\overrightarrow{V_A}$, $\overrightarrow{V_B}$ of the bodies A and B, then A become at rest, and the velocity vector of B with respect to A becomes.



$(\overrightarrow{V_B} - \overrightarrow{V_A})$ i.e.: $\boxed{\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}}$

Critical thinking: If $\overrightarrow{V_{BA}}$ is the velocity vector of B with respect to A , $\overrightarrow{V_{AB}}$ is the velocity vector of A with respect to B, then write the relation between $\overrightarrow{V_{BA}}$, $\overrightarrow{V_{AB}}$

Example

(6) A car moves on a straight road with velocity 80 km/hr. A motorcycle moves on the same road with velocity 50 km/hr. Find the velocity of the motorcycle relative to the car If:

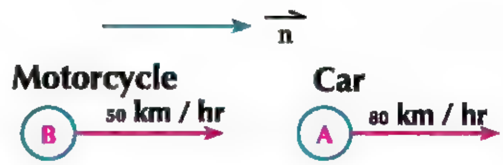
a The motorcycle moves in the same direction of the car.

b The motorcycle moves in opposite direction to the car.

Solution

Let the car be A and the motorcycle be B, and $\overrightarrow{n}$ be the unit vector in the direction of the car.

a The motorcycle (B) moves in the same direction of the car (A):



$\overrightarrow{V_B} = 50\ \overrightarrow{n}$, $\overrightarrow{V_A} = 80\ \overrightarrow{n}$, The velocity of the motorcycle with respect to the car $\overrightarrow{V_{BA}} = ?$

$\because \overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$ $\quad\therefore \overrightarrow{V_{BA}} = 50\ \overrightarrow{n} - 80\ \overrightarrow{n} = -30\ \overrightarrow{n}$

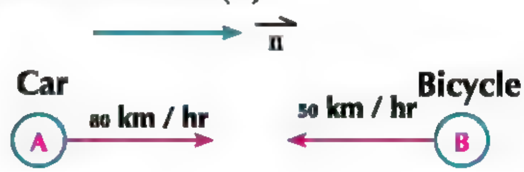
i.e. The motorcycle seems to the driver of the car as it moves away of the car with velocity of magnitude 30 km/hr in the opposite direction of $\overrightarrow{n}$.

b The motorcycle (B) moves in the opposite direction of the car (A):

$\overrightarrow{V_B} = -50\ \overrightarrow{n}$, $\overrightarrow{V_A} = 80\ \overrightarrow{n}$,

$\because \overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$

$\therefore \overrightarrow{V_{BA}} = -50\ \overrightarrow{n} - 80\ \overrightarrow{n} = -130\ \overrightarrow{n}$



i.e. The motorcycle seems to the driver of the car as it moves towards him with velocity of magnitude 130 km/hr.

Try to solve

(9) A car moves on a straight road with velocity 72 km/hr. A motorcycle moves on the same road with velocity 28 km/hr. Find the velocity of the motorcycle relative to the car If:

a The motorcycle moves in the same direction of the car.

b The motorcycle moves in opposite direction of the car.

Example

(7) A ship is moving in a straight way towards a port at 100 km. Far from it, a guard airplane passes over the ship in the opposite direction with velocity 250 km/hr. The guard airplane observes the ship, which seems to be moving with velocity 300 km/hr. Find the time from the moment of observation till it reaches the port.

Solution

Let the ship be B and the guard airoplane be A and let $\overrightarrow{n}$ be the unit vector in the direction of the velocity vector of the guard airoplane (A). And the actual velocity of the ship $\overrightarrow{V_B}$ (in opposite direction of the motion of the guard airoplane).

$\therefore \overrightarrow{V_A} = 250\ \overrightarrow{n}$, $\overrightarrow{V_{BA}} = -300\ \overrightarrow{n}$

$\because \overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$ $\therefore -300\ \overrightarrow{n} = \overrightarrow{V_B} - 250\ \overrightarrow{n}$

i.e.: $\overrightarrow{V_B} = -50\ \overrightarrow{n}$

i.e. The magnitude of the actual velocity of the ship equals 50 km/hr, which act in the opposite direction of the motion of the guard airoplane.

$\because \overrightarrow{S} = \overrightarrow{V}t$ $\therefore 100 = 50t$

i.e. t = 2 hours

Try to solve

(10) A moving radar car to monitor the velocity on the desert road moves with velocity 40 km/hr. This car observes the movement of a truck coming in the opposite direction. It seems like it is moving with velocity 120 km/hr. What is the actual velocity for the truck?

Exercise (2 - 1)

Complete the following:

(1) 20m/sec = ... km/hr

(2) 90 km/hr = ... m/sec

(3) A car moves with a uniform velocity of magnitude 72 km/hr for a quarter of an hour, then the covered distance = ... km.

(4) If $\overrightarrow{V_A} = 15\ \overrightarrow{i}$, $\overrightarrow{V_B} = 22\ \overrightarrow{i}$ $\therefore \overrightarrow{V_{BA}} =$ ....

(5) If $\overrightarrow{V_{AB}} = 65\ \overrightarrow{n}$, $\overrightarrow{V_A} = 50\ \overrightarrow{n}$ $\therefore \overrightarrow{V_B} =$ ....

(6) A cyclist (A) moves on a straight road with a velocity of 15 km/hr. If he met another cyclist (B) moves with a velocity of 12 km/hr, then the velocity of B with respect to A equals .... km/hr.

Choose the correct answer:

(7) If a car moves with uniform velocity 75 km/hr for 20 minutes, then the covered distance equals .... km

a) 15 b) 20 c) 25 d) 30

8) A car covered a distance of 180 km. With velocity 20 m/sec on a straight road, then the time taken to cover this distance = ... hours

a) $1\frac{1}{2}$ b) - 2   c) $2\frac{1}{2}$ d) 3

9) If $\overrightarrow{V_{AB}} = 15\ \overrightarrow{i}$, $\overrightarrow{V_A} = 35\ \overrightarrow{i}$ $\therefore\ \overrightarrow{V_B}$ equals:

a) $-50\ \overrightarrow{i}$ b) $-20\ \overrightarrow{i}$ c) $20\ \overrightarrow{i}$ d) $50\ \overrightarrow{i}$

10) If the position vector of a particle moves in a straight line from a point and gives a function in time t by the relation: $\overrightarrow{r} = (2t^2 + 3)\ \overrightarrow{n}$ then the magnitude of position vector $\overrightarrow{S}$ its norm is measured by meter after 2 seconds equal:

a) 4m   b) 6m   c) 8m   d) 11m

11) **Join with space:** If the sun light reaches the earth in 8.3 min. and the distance between the earth and the sun equals 1.494×10^{11} meter , find the velocity of the light.

12) Two cars moves at the same time from Banha towards Cairo with a constant velocity for each of them. If the velocity of the first car equals 70 km\hr and the velocity of the second car equals 84 km\hr .Find the taken time by the driver of the second car to reach the first car at the end of the trip whose length 49 km?

13) A train of length 150 meter entered a straight tunnel of length S meter. It took the entire crossing of the tunnel in a time of 15 seconds. Find the length of the tunnel if the train moves with uniform velocity equals 90 km/hr.

14) A cyclist covered 30 km on a straight road with a velocity of 15 km/hr, then he returned back and covered 10 km in the opposite direction with a velocity of 10 km/hr. Find the average velocity during his whole trip.

15) A traveller moved on a straight road, he covered 800 meters with velocity 9 km/hr, and then he returned back and covered the same distance in the same direction with velocity 4.5 km/hr., Find the magnitude of the average velocity of the traveller during the whole trip.

16) The distance between two cities A and B is 120 km. A car moved from the city A towards the city B with a velocity of 88 km/hr. At the same moment , another car moved from the city B towards the city A with a uniform velocity of 72 km/hr. Find when and where do the two cars meet.

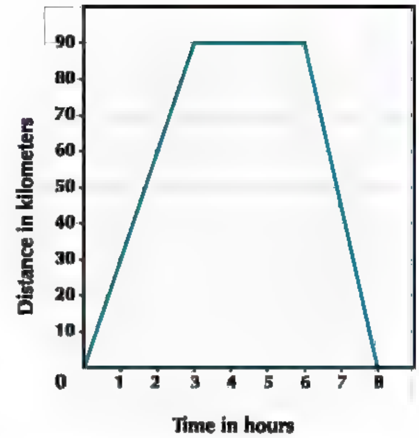
17) A car (A) moves on a straight road with a uniform velocity 60 km/hr., If another car (B) moves with a uniform velocity of 90 km/hr. on the same road. Find the velocity of car (A) relative to the car (B) if:

a) The two cars are moving in opposite direction.

b) The two cars are moving in the same directions.

18) A police car moves in a straight line with a uniform velocity, if it recorded the relative velocity of a truck moves in its direction in front of it which equals 60 km/hr. ,If the police car doubles its velocity, and recorded the relative velocity of a truck again which seems to be at rest.   Find the real velocity of each of the police car and the truck.

Activity (1)



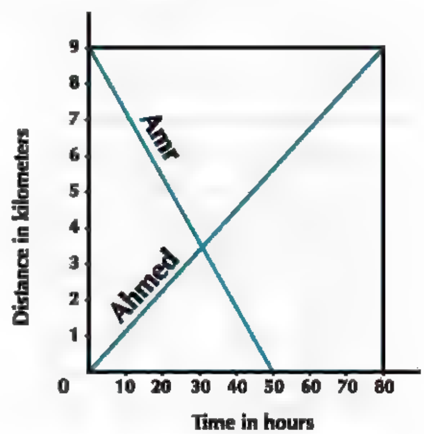
19 The opposite figure shows the relation between the distance in km, and the time in hours for the path of motorcycle moves between two cities. Answer the following:

- **a** Find the average velocity for the motorcycle during travelling?
- **b** Find the average velocity for the motorcycle during returning?
- **c** What is the description of the horizontal line segment shown in the opposite figure?

20 A motorcycle moves with a uniform velocity , then after 1 minute it becomes at a distance 2 km from the point A, and after 3 minutes it becomes at a distance 5 km from the same point. Draw a graph represents the relation between the distance and the time for this motorcycle, and from the graph:

- **a** Find the velocity of the motorcycle?
- **b** Write the mathematical relation between the time (t) and the distance (s).

Activity (2)



21 The opposite figure shows the path of movement of each of Ahmed and Amr in covering the distance between two villages, One of them is in the first village, while the other is in the second Village.

- **a** Do Ahmed and Amr start moving at the same time? Explain
- **b** After how many minutes did Amed and Amr meet?
- **c** What is the time taken by Ahmed to cover the distance?
- **d** Find the velocity of Amr.
- **e** If Amr starts moving at 9 : 30 am , so when does he reach the other Village?

22 If the position vector of a particle moving in a straight line from the point O is $\overrightarrow{r}$ and it is given a function in time by the relation : $\overrightarrow{r} = (t^2 + 3t - 2)\ \overrightarrow{i}$ where $\overrightarrow{i}$ is a fixed unit vector. Find the displacement vector after 4 seconds.

23 A particle has been found at two moments 3, 8 seconds at the two positions A(4, 3), B (12,9) respectively. Find the direction of the average velocity of the particle during this period of time, then find the magnitude of this average velocity.

24 **Creative thinking:** A man walks on a bridge AB, and when he covers $\frac{3}{8}$ of the bridge length from the point A he heard a beep sound of a train moves behind him with a uniform velocity 60 km/hr. towards the point A. If the man moves towards the train, Then the train will hit him directly at the point A. Find the lower uniform velocity that the man must moves with , before getting crushed by the train directly at the point B.

2 - 2 Rectilinear motion with Uniformly accelerated

You will learn

- Acceleration
- Time - velocity curve
- Uniformly variable motion
- Relation between velocity - time
- Relation between distance - time
- Relation between velocity - distance

Key terms

- Acceleration
- Uniform variable motion
- Uniform acceleration
- Uniform deceleration

- Squared paper.
- Scientific calculator.
- Drawing computer programs.

Introduction:

We have studied the regular motion in a straight line with regular speed. It is observed that a small number of objects move this way for a long time, and that each car has three tools to control its speed. They are the gas pedal, brake pedal and the steering wheel which controls the direction of movement of the car. We also observe the change in objects, velocities during their falling and during their projecting upwards.

Learn

Def.

Rectilinear Uniformly accelerated motion: it is the motion occurred by the change of the magnitude of the velocity regularly by time and it is called acceleration. Such that :

$$\text{Acceleration (a)} = \frac{\text{final velocity - initial velocity}}{\text{time}}$$

Measuring units: m/sec^2 **or** cm/sec^2 **or** km / hr^2

Notice that:

The change in the velocity at a specific instance of time is called the instantaneous acceleration.

Velocity-Time curve

The concept of acceleration associated with the change in velocity, so if the value of velocity increases with time we say: the motion is accelerated, and the acceleration is positive (as velocity is positive) as in figure (1).

And if the value of the velocity decreases with time we say: The acceleration is negative as in figure (2).

And if the velocity is constant with time we say: the motion is uniform (regular).

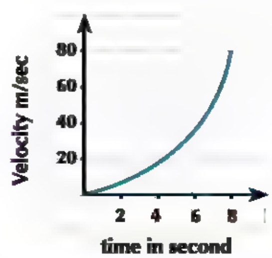


Fig (1)

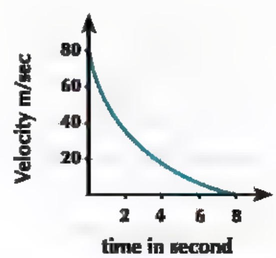


Fig (2)

Uniformly accelerated motion

It was said that the particle moves in a uniformly accelerated motion or with uniformly acceleration if the acceleration vector is constant in magnitude and direction at all times.

Verbal expression: What do the following statements mean?

- **a** The magnitude of the velocity of a particle increases during its motion a uniformly increasing by the rate 4 m/sec^2.
- **b** The magnitude of the velocity of a particle decreases during its motion a uniformly decreasing by the rate 24 km/hr^2.

Example

(1) If the velocity of a moving car in a straight line changes from 50 km/hr to 68 km/hr during 10 seconds. If a truck has been moved from rest, till its velocity becomes 18 km/hr during this period. Which of them move with higher acceleration? Explain your answer.

Solution

From the data, it is shown that both of them (the car, the truck) have an increase in velocity with magnitude 18 km/hr (i.e. 5 m/sec) during a period of time equals 10 sec. which means that the acceleration is equal for each of them.

then: The acceleration that each of them move with is:

$$\because\ a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{5}{10} = \frac{1}{2}\ m / sec^2$$

Try to solve

(1) If the velocity of a moving car(A) in a straight line changes from 24 km/hr into 36 km/hr during 5 seconds, and the velocity of another car (B) moves in the same straight line changes from 12 km/hr into 30 km/hr during the same period of time. Which of them moves with higher acceleration? Explain your answer.

Equations of the uniform variable motion in a straight line

There are three basic equations link between the algebraic magnitude for the vectors of displacement, velocity, acceleration and time in the case of moving with uniform acceleration and they are:

First: the relation between the velocity and time:

If a particle moves in a straight line by an initial velocity vector $\overrightarrow{V_0}$, fixed acceleration vector $\overrightarrow{a}$ and its velocity vector become $\overrightarrow{V}$ after an interval of time (t) then :

$$\overrightarrow{a} = \frac{\overrightarrow{V} - \overrightarrow{V_0}}{t} \quad \text{i.e.:} \quad \boxed{\overrightarrow{V} = \overrightarrow{V_0} + \overrightarrow{a}\ t}$$

by taking the algebraic measurement it will be $\boxed{V = v_0 + a\,t}$

Note that:

- From the relations between four unknowns, we can find one of them by knowing the other three.
- If the body started its motion from rest , So $v_0 = 0$, then $V = at$
- If $a = 0$ then $V = V_0$ i.e. the body moves with uniform velocity.

Example

(2) A particle starts its motion in a constant direction with velocity 9 cm/sec and with a uniform acceleration of magnitude $3 cm/sec^2$ acts in the same direction of the initial velocity. Find:

a The velocity of the particle after 5 seconds from the starting of the motion.

b The time it takes from the starting of the motion until its velocity become 54 cm/sec.

Solution

a We assume that the positive direction is the direction of the motion of the particle.
From the givens of the problem: $v_0 = 9cm/sec$, $a = 3cm/sec^2$, $t = 5$ seconds.
$\because\ V = v_0 + a\,t \qquad \therefore V = 9 + 3 \times 5 \qquad \therefore\ V = 24$ cm/sec.

b $\because\ V = v_0 + a\,t \qquad \therefore\ 54 = 9 + 3\,t \qquad \therefore\ t = 15$ seconds.

Try to solve

(2) A particle starts it motion in a constant direction with velocity 20 cm/sec and with a uniform acceleration $5 cm/sec^2$ acts in the same direction of vector of the initial velocity. Find:

a Its velocity at the end of one minute from the starting of the motion.

b The taken time from the starting of the motion until its velocity becomes 18 km/hr.

Example

(3) A particle moves in a straight line so, its velocity changes from 54 km/hr into 3 m/sec in half a minute. Find the magnitude of the acceleration of the motion. Can this particle be at rest instantly? Explain your answer.

Solution

To transfer the velocity from km/hr into m/sec

$54\ km / hr = 54 \times \frac{5}{18} = 15\ m / sec$

"From the given of the problem" $v_0 = 15\ m / sec$, $V = 3m / sec$, $t = 30$ seconds .

$\because\ V = v_0 + a\,t \qquad \therefore 3 = 15 + 30\,a$

i.e.: $30\,a = -12 \qquad \therefore a = -0.4\ m/sec^2$

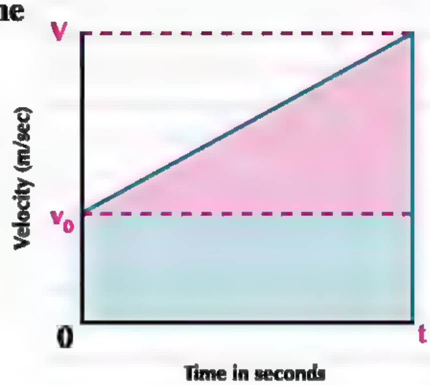
$\because a < 0$ The particle can rest instantly because it moves with a deceleration motion.

Try to solve

(3) A car moves in a straight line so its velocity decreases from 63 km/hr into 36 km/hr in a time of magnitude half minute. Find the acceleration that the car moves with and the time that it takes after that until it comes at rest instiniously.

Second: The relation between displacement and time

The area under the curve (The velocity - the time) equals the displacement of the body. In the opposite figure: The body moves with uniform acceleration starting with initial velocity V_0 and after time t second , its velocity becomes V . The area under the curve can be calculated by dividing it into a rectangle and a triangle.



Area (S) = area of rectangle + area of triangle

$$= v_0 t + \frac{1}{2} t (v - v_0)$$

$$S = v_0 t + \frac{1}{2} t (v_0 + a t - v_0)$$ (and that by substituting from the first law: $v = v_0 + a t$)

$$\boxed{S = v_0 t + \frac{1}{2} a t^2}$$

where: S, V_0 , a are the algebraic measurment to the displacement, velocity, acceleration vectors.

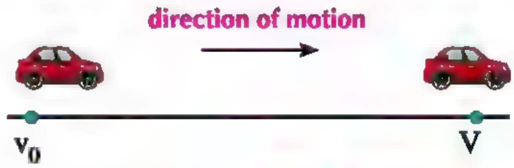
Verbal Expression:

1- Write the formula of the law (The distance - the time) when the body starts its motion from rest.

2- Write the formula of the previous law when a = 0 , and mention the type of the motion in this case?

Example

(4) A car moves with velocity 90 km/hr, the driver pressed on the brake, where the velocity decreases with constant rate until the car stopped after 5 seconds. Calculate:



a Acceleration of the car during the decreasing of the velocity.

b The distance that the car covered until its motion stopped completely.

Notice

When the body stopped, then V = 0

Solution

a To convert the velocity from km/hr into m/sec : 90km/hr = $90 \times \frac{5}{18}$ = 25 m/sec

by substitution in the law: $V = v_0 + a t$ where $v_0 = 25$ m/sec , V = 0 , t = 5 seconds

$\therefore 0 = 25 + 5 a$ i.e. $a = -5$ m/sec^2

i.e. The car moves with uniform deceleration of magnitude 5 m/sec^2.

b $\because S = v_0 t + \frac{1}{2} a t^2$ by substitution : $v_0 = 25$m/sec, $t = 5$sec , $a = -5$m/sec^2

$\therefore S = 25 \times 5 + \frac{1}{2}(-5) \times 25 = 62.5$ meters.

Try to solve

(4) A small ball is projected horizontally with velocity 20 m\sec. then it moves in a straight line with a retardation motion by a uniform acceleration $\frac{1}{2}$ m\sec^2. Determine the position of the point and its velocity after 2 sec from the starting of the motion.

Third: The relation between the displacement and velocity

We know that: $V = v_0 + a t$ **(1)** $S = v_0 t + \frac{1}{2} a t^2$ **(2)**

By squaring the first equation: $v^2 = V_0^2 + 2 v_0 a t + a^2 t^2$.

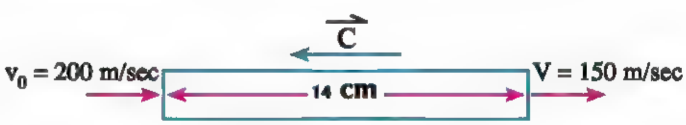
$\therefore v^2 = V_0^2 + 2a(v_0 t + \frac{1}{2} a t^2)$ By substitution from the equation (2) by the value of S

$$\boxed{v^2 = V_0^2 + 2 a s}$$

Example

(5) A bullet was fired with velocity 200 m/sec in perpendicular direction on a vertical wall with a thickness of 14 cm. So, it went out with velocity 150 m/sec. Find the magnitude of the deceleration, and if the bullet was fired with the same velocity on another identical vertical wall with the same resistance. So, Find the distance that it embedded until it rests, If it is known that the acceleration that the bullet moves with it is the same in both cases.

Solution



We assume the positive direction is the direction of motion of the bullet.

The first case: $v_0 = 200$m/sec, $V = 150$ m/sec , $S = 0.14$ m

$\because v^2 = V_0^2 + 2 a s$ $\therefore (150)^2 = (200)^2 + 2 \times C \times 0.14$

and by simplification: $a = -62500$ m/sec^2

The second case:

$v_0 = 200$m/sec , $V = 0$ $C = -62500$ m/sec^2

$\because v^2 = V_0^2 + 2 a s$ $\therefore 0 = (200)^2 - 2 \times 62500\, s$

$\therefore s = 0.32$ meters i.e. the bullet embedded in the wall a distance 32 cm until it comes to rest.

Try to solve

(5) The velocity of the car decreases uniformly from 45 km/hr into 18 km/hr after it covered a distance 625 meters. Find the distance that it covers after that until it comes to rest.

(6) A bullet was fired horizontally on a wooden block with velocity 100 m/sec. So, it embedded in it a distance 50 cm. Find the acceleration that the bullet moves with it if it is an uniform acceleration and if it is fired on an identical wooden block for the first time its thickness 18 cm . What is the velocity that the bullet went out with it from the wooden block?

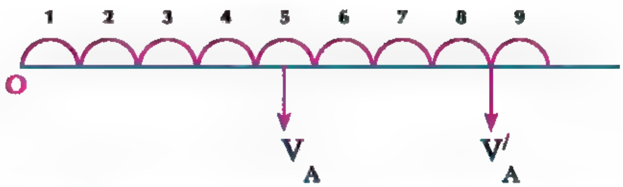
Example The average velocity within n^{th} second:

(6) A particle moves with an initial velocity 10 cm\sec in a constant direction and with uniform acceleration 4cm\sec^2 .Find:

First: The covered distance in the 5^{th} second of the motion.

Second: The covered distance in both of the 8^{th} and the 9^{th} seconds together.

Solution



The positive directions consider is the direction of the initial velocity so $\therefore v_0 = 10$ cm/sec , $a = 4$cm/sec^2

First: The average velocity V_A through the 5^{th} second = the velocity in the middle of the time interval = the velocity after $4\frac{1}{2}$ second.

$\because V_A = V_0 + a\,t$ $\qquad \therefore V_A = 10 + 4 \times 4\frac{1}{2} = 28$cm/sec.

The covered distance in the 5^{th} second = average velocity × time = 28 × 1 = 28 cm.

Second: The average velocity V_A through the 8^{th} and 9^{th} second = the velocity in the middle of the time interval = the velocity after 8 second from the start of the motion.

$\because V_A = V_0 + a\,t$ $\qquad \therefore V_A = 10 + 4 \times 8 = 42$ cm/sec

The covered distance in the 8^{th} and 9^{th} second = average velocity × time = 42 × 2 = 84 cm

Think:

Try to solve the previous example by another method.

Try to solve

(7) A particle started its motion in a constant direction with velocity 30 cm/sec and uniform acceleration 6 cm/sec^2 in the same direction of its velocity. Calculate:

a The distance covered after 5 seconds from the starting of the motion.

b The distance covered in the 5^{th} second only.

(8) A particle moves with an initial velocity in a constant direction and with uniform acceleration, if it covered in the 3^{rd} second from its motion a distance of 20 meters, then it covered in the two seconds, both the 5^{th} and 6^{th} a distance of 60 meters. Calculate the acceleration that the particle moves with it and its initial velocity.

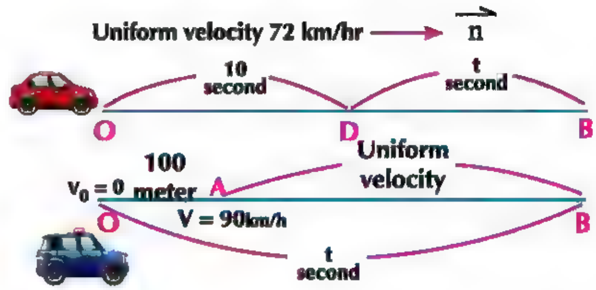
(9) A metro moves in a straight line between two stations A, B the distance between them is 700 meters, where it starts from the station A from rest with a uniform acceleration 2 m/sec^2 for 10 seconds ,then it moves after that with a uniform velocity in an interval of time, then it covered a distance of the last 60 meters from its motion with a uniform deceleration until it stops in the station B. Find the time taken in covering the distance between the two stations.

Example Applications on the laws of motion with uniform acceleration

(7) A car moves with a uniform velocity 72 km/hr passed by a police car, so the police car started following it after 10 seconds from its passing moving with uniform acceleration a distance of 100 meters until its velocity reached 90 km/hr , then it moves with that velocity until it reached the first car. Find the time taken through the chasing process starting from the of motion of the police car and the distance covered by the first car.

Solution

We consider the positive direction is the direction of the uniform velocity and the police car was at rest at point (0), then it covered a distance 100 meters until it reached point (A) where its velocity became 90 km/hr ,then it moved uniformly until it reached the first car at (B).



$72 \text{ km/hr} = 72 \times \frac{5}{18} = 20 \text{ m/sec}$, $90 \text{km/hr} = 90 \times \frac{5}{18} = 25 \text{ m/sec}$

With respect to the police car in the interval from $O \longrightarrow A$

$v_0 = 0$, $V = 25 \text{ m/sec}$, $S = 100 \text{ m}$ $v^2 = v_0^2 + 2as$

$25 \times 25 = 2 \times a \times 100$ $\therefore$ $a = \frac{25}{8} \text{ m/sec}^2$

$V = v_0 + at$ $\therefore$ $25 = \frac{25}{8} t$ $\therefore$ $t = 8$ seconds

$\therefore$ The distance that the police car moved on with uniform velocity = 25 (t - 8) meters

The chased car covered the distance and reached point B in time of magnitude = (t + 10) seconds

The police car covered the same distance and reached point B in time of magnitude = t second

$\therefore$ $20(t + 10) = 100 + 25(t - 8)$, then $t = 60$ second

The covered distance = $20 \times 70 = 1400$ meters

Try to solve

(10) A car moves with a uniform velocity 54 km/hr passed by a police car, so the police car started following it after 30 seconds from its passing moving with uniform acceleration a distance of 200 meters until its velocity reached 72 km/hr , then it moves with that velocity until it reached the first car. Find the time taken through the chasing process starting from the motion of the police car and the distance covered by the first car.

Exercises (2 - 2)

1. Complete the following:

   a. A particle moves in a straight line from rest with uniform acceleration of magnitude $4 m/sec^2$ so, its velocity after 6 seconds from starting the motion = ... m/sec.

   b. The distance that the particle covered in a constant direction from rest with acceleration. of magnitude $5 cm/sec^2$ during a time of magnitude 4 seconds = ... cm.

   c. The average velocity for a particle moving with initial velocity v_0 and a uniform acceleration (a) through the sixth second from its motion = ...

   d. The average velocity for a particle moving with initial velocity V_0 and a uniform acceleration (a) through the seconds 7^{th}, 8^{th} and 9^{th} from the starting of the motion = ...

   e. A particle moves from the rest in a straight line with uniform acceleration. So, it covered 24 meters in the first four seconds from its motion , then the magnitude of its acceleration = ...

   f. A particle started its motion from rest in a straight line with uniform acceleration of magnitude $2 cm/sec^2$ so, it covered a distance 25 cm ,then its velocity at the end of this distance = ... cm/sec.

2. A car moves from rest with acceleration of magnitude $4 m/sec^2$. What is the distance that the car covered when its velocity became 24 m/sec?

3. A racing car moves in the track with velocity 44 m/sec then its velocity decreases with a constant rate until it becomes 22 m/sec through 11 seconds. Find the distance that the car covered through that time .

4. A particle moves in a straight line with a uniform acceleration so its velocity increased from 15 m/sec into 25 m/sec. after covering 125 meter .Find the time takes for that .

5. A cyclist moves with a uniform acceleration until its velocity became 7.5 m/sec through 4.5 seconds. If the displacement of the bicycle through the accelerating interval equals 19 meters, find the initial velocity for the bicycle.

6. Karim practices on riding the bicycle. His father pushes him to gain a constant acceleration of magnitude $\frac{1}{2} m / sec^2$ for 6 seconds and after that Karim rides the bicycle alone with the velocity gained for another 6 seconds before he falls on the ground. Find the distance that Karim will cover.

7. A cyclist descends from the top of a hill with a constant acceleration of magnitude $2 m/sec^2$. When he reaches the base of the hill, his velocity reaches 18 m/sec. then he uses the brakes to preserve this velocity for one minute. Find the total distance that the cyclist covered.

8. A car driver moves with a constant velocity of magnitude 24 m/sec. He suddenly saw a child passing the road. If the required time for the brakes to respond is $\frac{1}{2}$ sec then it moves with a uniform deceleration of magnitude $9.6 m/sec^2$ until it stopped. Find the total distance covered by the car before it stops.

9) A body started its motion from rest in a horizontal straight line with uniform acceleration of magnitude 4 cm/sec^2 for 30 seconds ,then it moves with the velocity it gained for another 40 seconds. Find the magnitude of its average velocity.

10) A body moves in a straight line with uniform acceleration on a smooth horizontal plane till it covered 26 meters through the 4th second from starting the motion and 56 meters through the 9th second only , Find its initial velocity and the magnitude of its acceleration.

11) x, y are two points on a horizontal straight road. The car (A) started the motion from x towards y starting from rest and with uniform acceleration 10 m/sec^2 and at the same moment another car (B) moves from y towards x with uniform velocity of magnitude 54 km/hr , if the relative velocity for the car (A) with respect to the car (B) at the moment of their meeting equals 162 km/hr ,find the time taken by each one of the two cars from the moment of their motion together until the moment of their meeting.

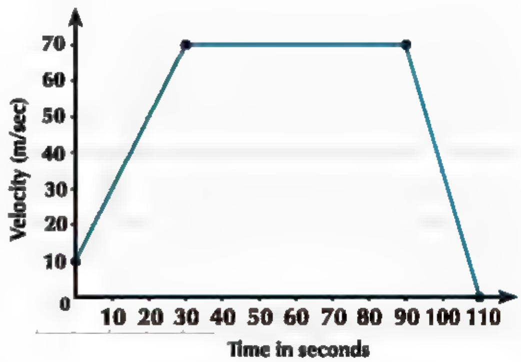
Activity

12) The opposite figure represents the curve (the velocity - the time) for a body started the motion with an initial velocity of magnitude 10 m/sec and until it came to rest after a time of magnitude 110 second. Find:

a The acceleration.

b The magnitude of the uniform deceleration for the body until it rests.

c The total distance that the body moves.

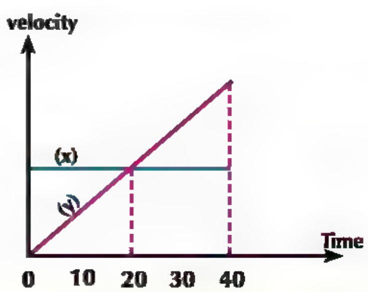


Creative thinking:

13) A lift is at rest at the bottom of a mine. The lift rises a distance 540 cm with acceleration of magnitude 120 cm/ sec^2, then it moves with uniform velocity for a distance 360 cm then with a uniform deceleration a distance 720 cm until it rests at the nozzle of the mine. Calculate the time that the lift takes in ascending from the bottom of the mine to its nozzle.

Creative thinking:

14) The opposite figure represents the curve (velocity – distance) for two cars x and y find the time taken by the two cars till they met.(Explain your answer)



Vertical motion under the effect of gravity (Free Fall)

2 - 3

Introduction:

What happens when an orange falls from a tree?

- The orange moves from rest, then it gains velocity during the free falling due to the effect of the Earth's gravity on it. After one second its velocity will be 9.8m/sec downwards and after another second its velocity will be 19.6m/sec downwards and so on.

Note that: The velocity of the orange is directly proportional with time. The acceleration that the bodies fall with in free falling (neglecting resistance of the air) equals 9.8 m/sec^2 approximately, and it denoted by (g) it varies by the variation of the latitude line so it decreases at the equatorial and increases slightly as we get closer to the poles. The acceleration will be positive or negative according to the coordinates system used, if the body fell or projected towards the surface of the earth then (g) , will be positive but if it projected upwards, then (g) will be negative.

Laws of the vertical motion of the bodies:

The vertical motion applies the same laws of the uniform motion with the uniform acceleration but with the usage of the symbol (g) instead of the symbol (a) for the acceleration when the bodies fall freely. Hence the laws will take the following forms:

$$V = v_0 + g\,t \quad , \quad S = v_0\,t + \frac{1}{2}\,g\,t^2 \quad , \quad v^2 = v_0^{\,2} + 2gS$$

where V, g, S are the algebraic magnitude of the vectors: velocity, acceleration and displacement.

So, when we apply the rules with the preceding form, we should notice V , v_0 , g , S according to the follows:

First: If the body is falling or projected downwards

We consider the vertical direction downwards is the positive direction. In this case, each of v_0 , V , g , S are positive.

Example

(1) A building worker dropped a piece of concrete from a high scaffold (platform).

a What is the velocity of the piece of concrete after half a second?

- Laws of vertical motion
- The study of the movement of the falling bodies or the projected downwards
- The study of the bodies projected upwards

- Free fall
- Acceleration of gravity

- Scientific calculator

b What is the distance covered by the mass building during this time?

Solution

a **Formula of the law:** $V = v_0 + g\,t$

by substitution $v_0 = 0$, $g = 9.8$ m/sec^2, $t = \frac{1}{2}$ sec.

$$V = 0 + 9.8 \times \frac{1}{2} = 4.9 \text{ m/sec}$$

b Formula of the law: $S = v_0\,t + \frac{1}{2}\,g\,t^2$

by substitution $v_0 = 0$, $g = 9.8$ m/sec^2, $t = \frac{1}{2}$ sec.

$$S = 0 + \frac{1}{2} \times 9.8 \times \frac{1}{4} = 1\frac{9}{40} \text{ meter.}$$

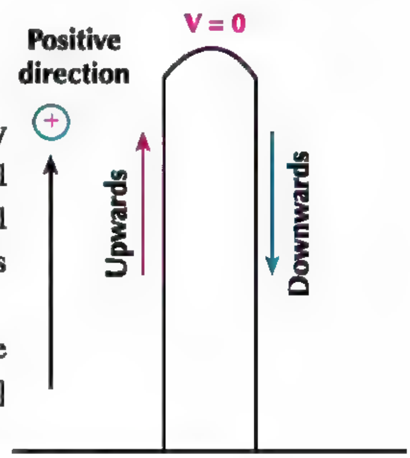
Try to solve

1. An apple fell from a tree and after one second it reached the ground.
   - **a** Calculate the velocity of the apple at the moment of reaching the ground surface, and then calculate the average velocity at the time taken to reach the ground surface.
   - **b** What is the distance of the apple from the ground surface at the moment of the beginning of its falling?

Second: If the body is projected vertically upwards

Activity

A ball is projected vertically upwards with initial velocity of magnitude 19.6 m/sec^2. By considering that the vertical direction upwards is the positive direction thus the initial velocity becomes positive but the acceleration becomes negative - **Why?**



- Use (geogebra) program in drawing the relation between (the velocity – the time) i.e. **$V = 19.6 - 9.8t$** when $t \in [0, 4]$

  What do you notice?

- Use the same program in drawing the relation between **(the distance - the time):**

  i.e. $S = 19.6\,t - 4.9\,t^2$, **What do you notice?**

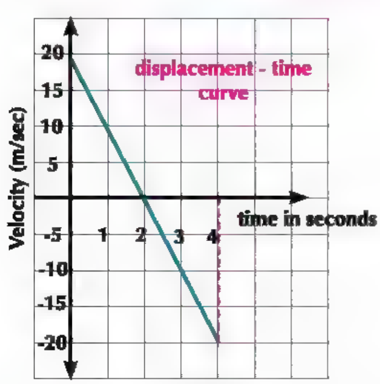


Fig (1)

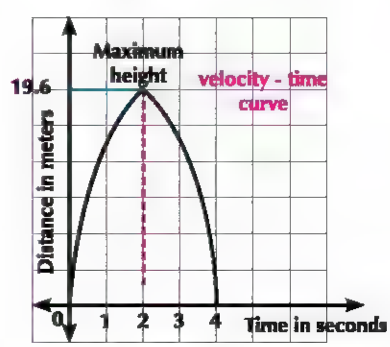


Fig (2)

We notice from the graph that:

- Velocity of the body during ascending is positive and during descending is negative.

  For example: when $t \in [0, 2[$ **we notice that velocity** $V > 0$, when $t \in]2, 4]$ So $V < 0$

- Velocity of the body at the point of maximum height equals zero.
- Time for ascending of the body equals time of descending.
- The magnitude of velocity of body when it returns to the projection point equals the magnitude of velocity of projection but with opposite sign.
- The displacement of the body during a time interval is not necessary to be equal to the distance covered by the body during this interval.

notice

The time of the maximum height = $\frac{v_0}{E}$

The maximum height = $\frac{V_0^2}{2E}$

Critical thinking:

1- If a body projected vertically upwards with initial velocity (V_0) and its final velocity reached (V) in time of magnitude (t) find.

a. The time taken by the body to reach the maximum height.

b. The maximum height that the body reaches.

Example

(2) If a body is projected vertically upwards with velocity 49 m/sec Find the time needed to reach the maximum height and the distance that it reaches.

Solution

Assume that the positive direction is vertically up ward :

$v_0 = 49 m/sec$, $g = -9.8 \ m/sec^2$, $V = 0$ (at the maximum height)

a. **To find the time needed to reach the maximum height:**

$\because V = v_0 + g\,t \qquad \therefore 0 = 49 - 9.8\,t. \qquad \therefore t = 5 seconds.$

b. **To find the distance of the maximum height:**

$\because v^2 = V_0^2 + 2\,gS \qquad \therefore 0 = (49)^2 - 2 \times 9.8 \times S \qquad \therefore S = 122.5$ meter

Think:

1- Can you use other laws to find the distance of the maximum height? (Explain that)

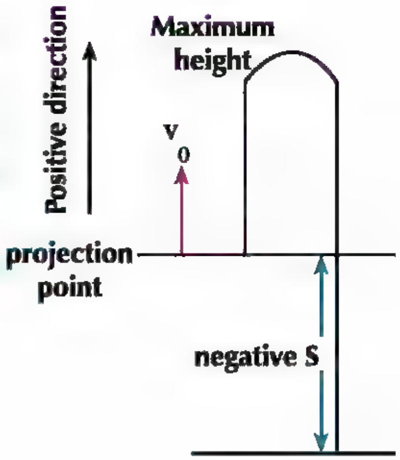
Try to solve

(2) A body projected vertically up wards with velocity 39.2 m/sec. Find the time for the maximum height and the maximum height the body can reach.

Example

(3) A body projected vertically upwards with velocity 16 m/sec. Find the time taken by the body to reach 330 meters under the projection point.

Solution



We consider the vertical direction upwards is the positive direction

$v_0 = 16$m/sec because it has the same direction of projection.

$g = -9.8$ because it is opposite to the direction of the Earth's gravity.

$S = -330$ because it is under (down) the point of the projection.

$S = v_0 t + \frac{1}{2} g t^2$

$-330 = 16t - \frac{1}{2} \times 9.8t^2$ by simplification $49t^2 - 16t - 330 = 0$

By factorization: $(t - 10)\ (49t + 330) = 0$

$t = 10$, $t = -\frac{330}{49}$ (Refused)

Think:

1- Do you have other solutions? Explain that.

Try to solve

(3) A small ball is projected vertically upwards from a window of a house and it was seen during its falling in front of the window after 3 seconds from its projection. It reached the ground surface after 4 seconds from the moment of projection. Find the height of this window over the ground surface.

Exercises (2 - 3)

(1) A child throws a ball from a window that rises 3.6 m from the pavement. What will be its velocity at the moment of contact with the pavement?

(2) A ball fell vertically downwards. What is its velocity after 6 seconds from the moment of its falling?

(3) A body fell vertically downwards from height 490 m from the surface of the ground find:

- a) Time of reaching the ground surface.
- b) Its velocity after 5 seconds from starting the motion.

(4) A rubber ball fell from a height of 10 meters so it hit the ground and rebounded vertically upward a distance $2\frac{1}{2}$ meters. Calculate the velocity of the ball just after and before hitting the ground.

(5) A student practices on kicking football vertically upwards in air, then the ball returns due to the impact of every kick. So, it hits his foot. If the ball takes from the moment of its kicking until colliding with his foot 0.3 seconds.

- a) Find the initial velocity.
- b) The height that the ball reaches after the student kicked it.

(6) A body is projected vertically upwards from the top of a hill of height 9.8 meters with velocity 4.9 m/sec Find:

- a Velocity of the body at the moment of reaching the bottom of the hill.
- b The time taken to reach the bottom of the hill.

(7) A stone is projected in a well with velocity 4 m/sec vertically downwards so, it reached the bottom of the well after 2 seconds. Find:

- a The depth of the well.
- b Velocity of the stone when it collides with bottom of the well.

(8) A particle is projected vertically upwards with velocity 14 m/sec from a point at height 350m from ground surface. Find the time that the particle takes to reach the ground surface.

(9) A ball is projected upwards from a window. So, it reaches it after 4 seconds from the moment of the projection and it reached the ground surface after 5 seconds from the moment of the projection. Find:

- a Velocity of the ball's projection.
- b Maximum height that the ball reached from the point of the projection.
- c The height of the window from the ground surface.

(10) A body is projected vertically upwards from the top of a tower its height 80.5 meters with velocity 8.4 m/sec. Find:

- a The maximum height that the body reaches from the point of the projection.
- b The time that the body takes while descending until its velocity become 11.2 m/sec.
- c The time taken by the body to reach the projection point.
- d The time taken by the body to reach the ground surface.

(11) A ball is projected from the top of the hill of height 140 m vertically upwards; it is found that it covered in the third second a distance 10.5 meters. Find:

- a The velocity that the ball is projected with.
- b The maximum height that the ball reached.
- c The time that the ball takes to reach the ground surface.

Creative thinking:

(12) A body fall from a height of 60 meters from the ground surface and at the same moment, another body is projected vertically upwards from the ground surface with velocity 20 m/sec. The two bodies meet after a time interval. Find this time, and then find the distance that the two bodies covered during this time interval. Mention whither the two bodies met each other moving in two opposite directions or in the same directions?

2 - 4 Newton's universal gravitation law

You will learn

- Newton's gravitational law
- The definition of the universal gravitational constant
- The comparison between the accelerations due to gravity on the surfaces of two planets

Key terms

- Universal gravitation
- Gravitational constant
- Gravitational force

- Scientific calculator

Think and discuss

What happens to the motion of the moon if the gravitational force between the moon and the Earth is lost? It will, confirmatory, take another orbit different from its elliptical orbit around the Earth.

Newton has realized that the force responsible for the gravitation of the Earth on the moon, and the gravitation of the sun on the planets is a special case of a universal gravitation between bodies.

You will know learn the universal gravitation law of Newton published in his mathematical research "principles of normal philosophy" in which he stated that:

All bodies in the universe are attracted to other bodies under the effect of a direct force which is directly proportion to their masses and inversely with the square of the distance between their centers.

If m_1, m_2 are two masses the distance between their centres is (S), then the value of the gravitational force between them is given by the relation:

$$\boxed{F = G \times \frac{m_1 \, m_2}{s^2}}$$ where

m_1, m_2 measured in kg, S in meters, G the universal gravitational constant.

Definition of the universal gravitational constant.

It is the gravitational force between two masses each of 1kg and the distance between them equals 1 meter and equals approximately 6.67×10^{-11} newton . m^2 / kg^2.

Verbal Expression:

1- Mention the factors affecting the gravitational forces between two bodies.

Think:

1- What happens to the gravitational force between two bodies if the distance between them increases?

2- Why does the gravitational force between the celestial bodies not appear obviously?

Example

1 Two balls , the first of mass 5.2 kg and the second of mass 0.25 kg, the two balls where put such that the distance between their centers equals 50 cm. Calculate the gravitational force between them, knowing that the universal gravitational constant equals 6.67×10^{-11} newton . m^2 / kg^2.

Solution

$m_1 = 5.2$ kg , $m_2 = 0.25$ kg , $S = \frac{1}{2}$ m , $G = 6.67 \times 10^{-11}$ newton.m^2 / kg^2

$F = G \times \frac{m_1 m_2}{S^2}$ $\quad \therefore F = \frac{5.2 \times 0.25}{\frac{1}{4}} \times 6.67 \times 10^{-11}$

$F = 3.4684 \times 10^{-10}$ newton (and it is a very small force)

Try to solve

1. If you know that the mass of the Earth equals 6×10^{24} kg the mass of the moon equals 7×10^{22} kg, the distance between them equals 3×10^{6} m and the universal gravitational constant equals 6.67×10^{-11} newton. m^2 / kg^2. Find the gravitational force of the Earth to the moon.

Example

2. A satellite of mass "m" kg revolves in an orbit of height 440 km from the Earth's surface whose mass equals 6×10^{24} kg and its radius 6360 km Find "m" to the nearest kilogram knowing that the universal gravitational constant equals 6.67×10^{-11} newton.m^2 / kg^2. if the gravitational force of the Earth on the moon is 17310 newton.

Solution

$m_1 = m$, $m_2 = 6 \times 10^{24}$, $S = (6360 + 440) \times 1000$ m substituting in the law: $F = G \times \frac{m_1 m_2}{S^2}$

$17310 = 6.67 \times 10^{-11} \times \frac{m \times 6 \times 10^{24}}{(6800 \times 1000)^2}$

i.e.: $m = \frac{17310 \times (6800 \times 1000)^2}{6.67 \times 10^{-11} \times 6 \times 10^{24}}$ $\qquad m = 2000.035982 \simeq 2000$ kg

(1 7 3 1 0 × (6 8 0 0 × 1 0 0 0) x^2) ÷ (6 . 6 7 × 1 0 $x^\blacksquare$ − 1 1 × 6 × 1 0 $x^\blacksquare$ 2 4 ►) =

Try to solve

2. A satellite of mass 1500 kg revolves at a height of 540 km from the Earth's surface whose mass is 6×10^{24} kg and radius is 6360 km.

Find the Earth's gravitational force on the moon knowing that the universal gravitational constant equals 6.67×10^{-11} newton . m^2 / kg^2

Example Calculating the Earth's mass

3. Calculate the mass of the Earth in kilograms assuming a body of mass 1 kg is put on its surface. Knowing that the length of the Earth's radius equals 6360 kg, $G = 6.67 \times 10^{-11}$ newton . m^2 / kg^2

Solution

The Earth's gravitational force on the body = m g (where m = 1kg , g = 9.8 m / sec^2)

$F = 1 \times 9.8 = 9.8$ newton.

The Earth's radius = 6360×1000 m , $G = 6.67 \times 10^{-11}$ newton . m^2 / kg^2

Apply the universal gravitational law: $F = G \times \frac{m_1 m_2}{S^2}$

$9.8 = 6.67 \times 10^{-11} \times \frac{1 \times m\ \text{Earth}}{(6360 \times 1000)^2}$

The Earth's mass (m) $= \frac{9.8 \times (6360 \times 1000)^2}{6.67 \times 10^{-11}} \simeq 6 \times 10^{24}$ kg

Add to your information

The Earth's force of gravity on a body of mass is mkg = $m \times 9.8 = 9.8\ m$

Critical thinking: Does the mass of the Earth change in the previous example if the mass of the body put on its surface equals 1000 kg? Justify your answer.

Try to solve

(3) Calculate the Earth's radius assuming that a body of mass 1 kg is put on its surface knowing that the Earth's mass equals 6×10^{24} kg and the universal gravitational constant equals 6.67×10^{-11} newton . m^2 / kg^2

Example

Determining the acceleration due to gravity (g)

(4) Calculate the acceleration due to gravity in m / sec^2 units for a body of mass 1kg put on its surface. Knowing that the Earth's mass equals 6×10^{24} kg, the Earth's radius equals 6360km

Solution

The Earth's force of gravitation on a body = m g $\therefore F = 1 \times g$ i.e. $F = g$

$\therefore F = G \times \frac{m_1 m_2}{S^2}$ $\therefore g = 6.67 \times 10^{-11} \times \frac{1 \times 6 \times 10^{24}}{(6360 \times 1000)^2}$ $g \simeq 9.89379\ m / sec^2$

Try to solve

(4) In the previous example, calculate the acceleration due to gravity in m / sec^2 units for a body of mass 100 kg put on its surface. What do you notice?

Activity

Comparing the accelerations due to gravities on the surfaces of two planets:

If g_1, g_2 the acceleration due to gravity for each planet, m_1, m_2 their mass in kg, r_1, r_2 their radii in meters respectively, then from the previous

it is possible to deduce the following relation: $\boxed{\frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}}$

Example

(5) If the mass of the Earth is 81 times the mass of the moon and their diameter equal 12756 km, 3476 km respectively. If the acceleration due to gravity on the Earth equals $9.8\ m / sec^2$ what is the acceleration due to gravity on the moon's surface?

Solution

Let the mass of the moon is (m) kg ,then the mass of the Earth equals (81 m)

$r_1 = 6378$ km , $r_2 = 1738$ km , $g_1 = 9.8$ m / sec^2 , $g_2 = ?$

$\because \frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}$ $\qquad \therefore \frac{9.8}{r_2} = \frac{81\ m}{m} \times (\frac{1738}{6378})^2$

Simplifying: $\qquad \therefore$ g moon $\simeq 1.63$ m / sec^2

Try to solve

(5) If you know that the mass of the Earth equals 5.97×10^{24} kg and it radius equals 6.34×10^{6} km, the mass of the moon equals 7.36×10^{22} and its radius equals 1.74×10^6 km, Find the ratio between the gravity on the moon's surface to the gravity on the Earth's surface.

Exercise (2 - 4)

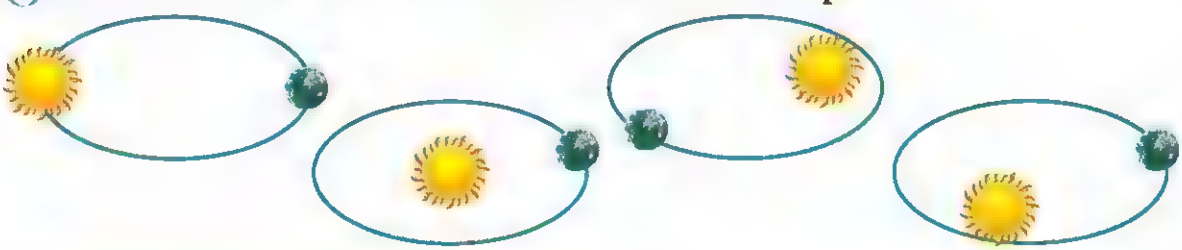
Note: let the Newton's universal gravitational constant: $G = 6.67 \times 10^{-11}$ N.m^2 / kg^2

(1) What happens to your weight when you go far from the Earth's surface?

(2) Why doesn't the gravitational force appear between the daily seen objects?

(3) What happens to the general gravitational force when the distance between them is doubled?

(4) Which of the shown orbits can be a suitable orbit for a certain planet:

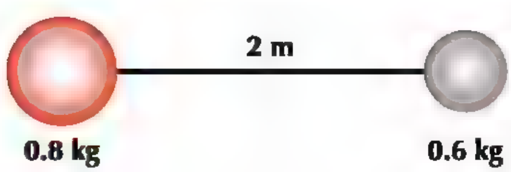


(5) **Choose the suitable answers:** A planet has two moons of equal masses, the first moon in a circular orbit of radius r, the second moon in a circular orbit of radius 2r, then the value of the gravitational force from the planet to the second moon is:

- a Four times the force affecting the first moon.
- b Two times the force affecting the first moon.
- c Equals the force affecting the first moon.
- d Half the force affecting the first moon.
- e Quarter the force affecting the first moon.

(6) **In the opposite figure:**

If the distance between the centers of the two balls is 2m, the mass of one of them is 0.8 kg and the mass of the other is 0.6 kg. What is the gravitational force between them?



7. Two identical balls each of mass 6.8 kg and the distance between their centers equals 21.8 cm. What is the gravitational force between them?
8. Calculate the gravitational force between two bodies of masses 10 kg , 15 kg and the distance between them is 2 meters.
9. A satellite of mass 2000 kg revolves at a height 440 km from the Earth's surface whose mass equals 6×10^{24} kg . Find the gravitational force of the Earth on the satellite knowing that the Earth's radius equals 6360 km.
10. If the Earth's acceleration due to gravity (g) is 10 m / sec^2 the Earth's radius equals 6.36×10^6 m. Calculate the Earth's mass.
11. Calculate the gravitational force between the Sun and the Earth if you know that the Earth moves in an elliptical orbit around the sun, the Earth's mass equals 6×10^{24} kg the sun's mass equals 9×10^{29} kg and the distance between their centers equals 1.5×10^{11} m.
12. If you know that the Earth's mass equals 5.97×10^{24} and its radius 6.34×10^6 m, the mass of the moon equals 7.36×10^{22} kg, find the radius of the moon if the gravity on the Earth's surface equals six times the gravity on the moon's surface.
13. If you know that the Earth's mass equals 6.06×10^{24} and its radius is 6.36×10^6 , find the intensity of the Earth's gravitational field.
14. A planet whose mass equals 3 times the Earth's mass, and its diameter equals 3 times the Earth's diameter. Calculate the ratio between the acceleration due to gravity on this planet and that on the Earth.
15. Find the universal gravitational force between two planets the mass of the first equals 2×10^{21} ton and the mass of the second equals 4×10^{25} ton and the distance between their centers equals 2×10^6 km.
16. A piece of iron is put at a distance of 50 cm from another piece of Nickel of mass 25 kg then the gravitational force between them became $10^{-8} \times 6$ N. What is the mass of the piece of iron approximated to the nearest integer number?
17. If a body of math m\ kg on a height s meter from the surface of the earth whose radius is r meter and mass m kg .find the value of the gravitational force that act on the body.
18. **Join with space:** an international space station with weight on the surface of the earth 421997.6 newton .find its weight when it became in an external orbit on a height 350 km from the surface of the earth known that the Earth's mass equals 6.37×10^3 and its radius is 5.6×10^{24} . (Hint : Force in Newton = Mass in kg × gravitational force of the earth 9.8 m/sec^2)

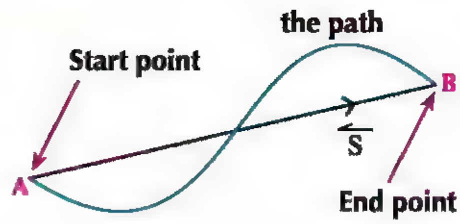
General Exercises

For more exercises please visit the website of the Ministry of Education.

Unit summary

The displacement vector

It is the vector represented by the directed line segment $\overrightarrow{AB}$ whose start point is (A) and end point is (B) the displacement vector $\overrightarrow{AB}$ is denoted by the symbol $\overrightarrow{S}$, and the norm of the displacement vector is denoted by the symbol $\| \overrightarrow{AB} \|$



The position vector

It is the vector whose start point is the point of the watcher (O) and its end point is the point of the body and it is denoted by the symbol $\overrightarrow{r}$

The relation between the position vector and the displacement vector:

$$\overrightarrow{S} = \overrightarrow{r} - \overrightarrow{r_0}$$

The velocity vector

The velocity vector of a body is the vector whose norm equals the value of the velocity and its direction is the same as the motion direction.

The uniform speed

It is the state at which each of the norm and the direction of the velocity vector are constant. i.e.: the body moves in a constant direction, where it covers equals distances in equal time intervals .

and the relation between the algebraic measures of the two vectors $\overrightarrow{S}$, $\overrightarrow{V}$ in case of the uniform velocity is: $\boxed{S = V\,t}$

The vector of the average velocity

If a body moves at two instances of times t_1 , t_2 by two positions A , B respectively, and $\overrightarrow{S}$ is the displacement vector in the time interval $(t_2 - t_1)$ then $\overrightarrow{V_A}$ is defined as the average velocity vector of this body during this time interval and:

$$\overrightarrow{V_A} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1} = \frac{\overrightarrow{S}}{t_2 - t_1}$$

The instantaneous velocity:

If a body moves with a variable velocity according to the curve (distance - time), then the slope of the tangent to the curve at a certain point on the curve at a certain time is known as the instantaneous velocity.

The relative velocity:

The relative velocity of a body (A) relative to another body (B) is the velocity with which the body (B) will appear to be moving if we considered the body (A) in a state of rest, considering $\overrightarrow{V_A}$, $\overrightarrow{V_B}$ as the velocity vectors of the two bodies A and B and the vector of the velocity of B with respect to

A is $\overrightarrow{V_{BA}}$ then $\boxed{\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}}$

The straight variable motion:

It is the motion in which the value of the velocity is changed with time, called the acceleration and its unit is m / sec^2.

$$\text{The acceleration (a)} = \frac{\text{Terminal velocity- initial velocity}}{\text{Time}}$$

The uniformly changing motion:

The body is called to be moving with a uniformly changing motion or with a uniform acceleration if the acceleration vector is constant in its value and direction all time.

If a body moves in a straight line with an initial velocity (Vo) and a constant acceleration (a) after a time interval (t) it cuts a distance (S), then :

- The relation between velocity and time: $V = v_0 + a\,t$
- The relation between distance and time: $S = v_0\,t + \frac{1}{2}\,a\,t^2$
- The relation between velocity and distance: $v^2 = v_0{}^2 + 2\,as$

it is noticed that each relation link four variables one of which could be determined knowing the other three variables.

- The area under the velocity-time curve equals the displacement of the moving body.
- The average velocity of a body during a certain time interval equals its instantaneous velocity at the middle of this time interval.

The laws of the vertical motion of bodies:

The vertical motion laws are the same as the laws of the straight motion with uniform acceleration but with using the symbol (g) which represents the free fall acceleration of the bodies instead of the symbol (a) such that the laws take the following forms:

$$V = v_0 + g\,t\,,\quad s = v_0 t + \frac{1}{2} g\, t^2\,,\quad v^2 = v_0^2 + 2gS$$

If a body is projected vertically upwards under the effect of the Earth's gravity and returned back to the point of projection , then:

- The velocity of the body is positive during its rise and negative during its fall.
- The velocity of the body at the maximum height equals zero.
- The rising time equals the falling time.
- The time of the maximum height (t) = $\frac{v_0}{g}$
- The maximum height the body can reach (S) = $\frac{V_0^2}{2\,g}$
- The value of the velocity by which the body returns back to the projection point equals the value of the projection velocity but with a different sign.
- The displacement of a certain body during a certain time interval is not necessarily equal to the distance covered by the body during the same time interval.

Newton's universal gravitational law

If (S) is the distance between two masses m_1, m_2 then the value of the gravitational forces between them is (F) and detrmine by the relation: $F = G \times \frac{m_1\, m_2}{s^2}$ where m_1, m_2 measured in kg, S in meters.

The universal gravitational constant:

It is the gravitational force between two masses each of 1 kg and the distance between their centers is 1 meter and it approximately equals 6.67×10^{-11} newton . m^2/kg^2

Enrichment Information

Please visit the following links.

Accumulative test

1. Two forces of magnitudes 8.16 newton act on a particle, Find:
   - a. The magnitude of their maximum resultant.
   - b. The magnitude of their minimum resultant.
   - c. The magnitude and direction of their resultant when the measure of the angle between them equals 120°.

2. The forces 12 , $5\sqrt{2}$, $2\sqrt{2}$, 8 gm wt act on a particle towards east , north of west, south west and south respectively. Find the value and direction of the resultant of these forces.

3. A body of weight (w) Newton is held by two string inclined to the vertical by two angles whose measures are θ° and 30° .If the body is in equilibrium when the tension in the first string equals 12 Newton and the tension in the second string is $12\sqrt{3}$ Newton .Find the values of θ° and the weight W.

4. A body of weight 90 kg is put on a plane inclined to the horizon by an angle of 30°. The body is kept in equilibrium by force acting on the body upwards in a direction inclined to the plane by an angle of measure 30°. Find the value of the force and the reaction of the plane.

5. A uniform rod $\overline{AB}$ its terminal (A) is linked to a hinge fixed on a vertical wall, a horizontal force acts on the other terminal (B) to keep it equilibrium when the rod inclined to the vertical by an angle 45°. If the weight of the rod equals 4 kg.wt ,Find the magnitude of the force and the value of the reaction of the hinge.

6. A police car (A) moves on a straight road with a velocity of 25 km/hr. It observes another car (B) moving on the same road with a velocity of 75 km/hr. Find the relative velocity of the car (B) with respect to the car (A) if:
   - a. The two cars are moving in the same direction.
   - b. The car (B) is moving in the opposite direction of car (A).

7. A particle moves in a straight line with a uniform acceleration 5 cm/sec^2 ,and in the same direction of the initial velocity of this body whose value equals 40 cm/sec. Find:
   - a. The values of the velocity and the displacement of it by the end of 24 seconds from the starting of the motion.
   - b. The value of the velocity of the body when it covered a distance 56 m from the starting point.

8. A car is moving on a straight road with uniform deceleration with magnitude 14 cm/sec^2, where it stopped after 20 seconds from its start. Find:
   - a. The value of its initial velocity.
   - b. The distance it covered in half a minute.
   - c. The distance it covered till it stopped.

9. A particle is projected vertically down from a point above the ground ,so it imbedded on the ground a distance 14 cm before it rests .If the particle moved inside the ground by uniform deceleration 63 m/sec^2 .find the height in which the particle fall.

10. A particle is projected vertically up from the top of a tower with velocity 24.5 m/sec .It reached the surface of the ground after 8 sec. Find:

   a. The height of the tower.

   b. The maximum height it can reach above the ground surface.

   c. The total distance covered by the particle in this period of time .

11. Which of the following represents the movement of a particle in a uniform motion :

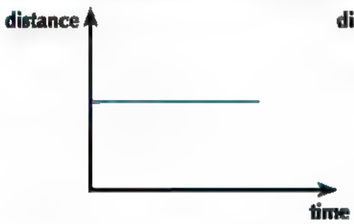


Fig (1)

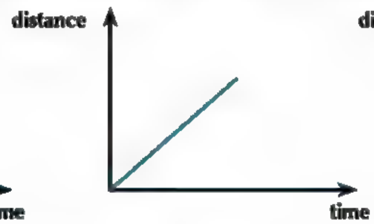


Fig (2)

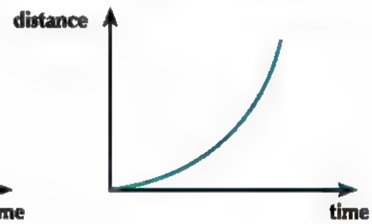
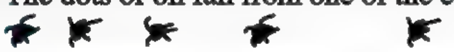


Fig (3)

12. The dots of oil fall from one of the cars moving from the left to the right as the figure :



By observing these dots then the car moves with:

1 - Uniform motion.

2 - positive acceleration.

3 - negative acceleration.

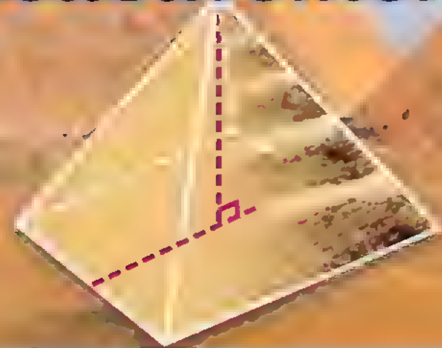
4 - negative acceleration then a uniform velocity.

If you cannot answer these questions you can use the following table:

If you cannot answer these questions	1	2	3	4	5	6	7	8	9	10	11	12	13
go back to	80	16	26	34	37	37	60	66	70	76	77	56	66

Unit Three

Geometry and Measurement



Introduction

Geometry originated in the beginning linking to the scientific aspect. It was used by the ancient Egyptians in determining the areas of land and construction of the pyramids and temples. They calculated areas of some shapes and volumes of some solids.

When Thales (640 - 546 B.C) went to Alexandria. He liked the ways in which the Egyptians measured the ground and name these ways by the word "Geo-metron" which is taken from the Greek language and consists of two words: the 1st word is Geo which means the land .the 2nd word is metron which means measurement. He was interested in studying Geometry as explicit expressions of abstract subject to proof.

Geometry evolved at the hands of the Greeks (Thales - Pythagoras - Euclid) by the emergence of a series of theories based on some axioms, definitions arranged in a logical accurate system in which included in Euclid's book The principles which consists of 13 parts.

Alexandria continued to be a beacon of knowledge till Arabs came and kept that heritage by translated it into Arabic and added many additions to it and transferred it to Europe in the twelfth century.

In the sixteenth century, Renaissance began in mathematics and the birth of a new science, Decarts (1650 - 1596) introduced the foundations of analytical geometry , the graphical representation of the equations , the express of the geometrical shapes by equations and deduced the equation of the circle $X^2 + y^2 = r^2$.

Euler also reached to the existence of a relationship between the number of faces and the number of edges of any solid has slatted base area, and it is: number of faces + number of heads = number of edges + 2.

Objectives of the unit

By the end of the unit the student should be able to:

- Define the point straight line and the plane in the space
- Recognize some solids such as: pyramid - the regular pyramid the right pyramid the cone - the right cone and the properties of each.
- Conclude the total surface area and the lateral area of each of the right pyramid and the right cone.
- Deduce the volume of each of the right pyramid the right cone .
- Find the equation of the circle in terms of coordinates of each of its center and the length of its radius
- Conclude the general form of the equation of the circle
- Determine the coordinates of the center of the circle and the length of its radius using the general form of the equation of the circle.
- Apply what he taught in Geometry in modeling mathematical situations.

Basic Terms

- The point
- Straight line
- Plane
- Space
- Vertex
- Base
- Axis
- Circle
- Center
- Radius
- Diameter
- Pyramid
- Cone
- Lateral face
- Lateral edge
- Height
- Slant height
- Regular pyramid
- Right pyramid
- Net of a pyramid
- Right circular cone
- Lateral area
- Surface area

Materials

- Scientific calculator
- Computer - Graphic programs
- Geometrical instruments

Lesson of the unit

Lesson (3-1):straight lines and the plane

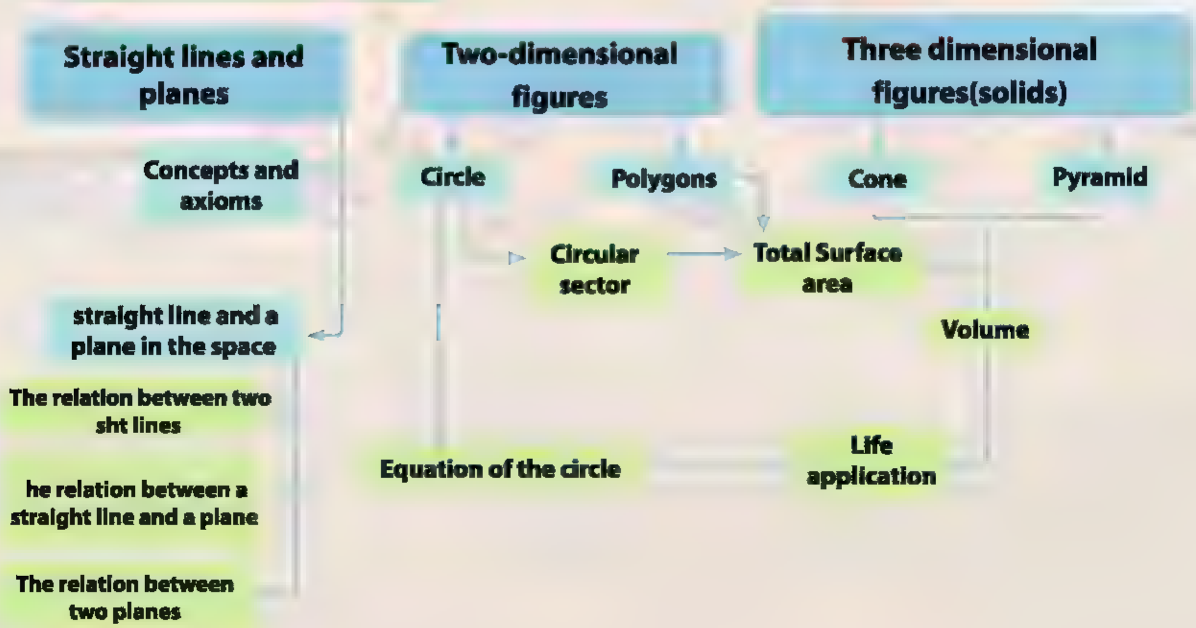
Lesson (3-2):Pyramid and cone

Lesson (3-3): Lateral area and total surface area of a pyramid and a cone

Lesson (3-4):Volume of a pyramid and a cone

Lesson (3-5):equation of a circle

Chart of the unit



3 - 1 The straight lines and the planes in a space

We will learn

- Concepts and geometrical axioms
- The relation between two straight lines in the space
- The relation between a straight line and a plane in the space
- The different positions of two planes in the space

Key - term

- Point
- Straight line
- Plane
- Space.

Materials

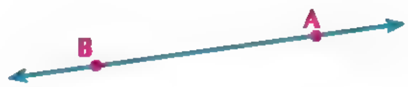
- Geometrical instrument
- Scientific calculator.
- Drawing programs.

Think and discuss

You have studied some mathematical concepts about the point, the straight line and the plane. Can you answer the following questions?

- How can you represent your city on the map of the Arab republic of Egypt?
- How many points are needed to draw a straight line?
- What does the following things related to you: the floor of the classroom , the surface of the table and the surface of the wall
- What does the following things related to you: the surface of the ball, the surface of the dome of the mosque and the surface of the cylinder of gaz.

Activity



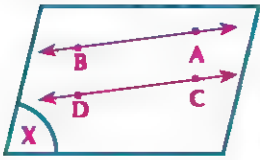
Draw two different points A and B on a paperboard.
Use the ruler to join the two points and extend them from each side.
Try to draw another straight line passes through the same points A and B, Can you do that?
What did you deduce from this activity?

Activity

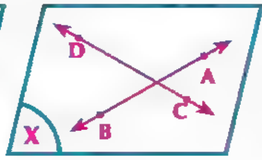
Draw three non-collinear points A, B and C as shown in the figure.
Use a paperboard in a form of a rectangle such that one of its dimensions lies on $\overleftrightarrow{AB}$ Move the plane of the paper so it rotate about $\overleftrightarrow{AB}$ to be on point C.
In how many position does the point C lies on the surface of the paper if the paper completes one turn?

Geometrical axioms:

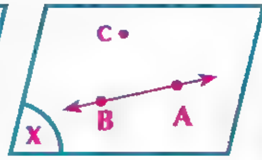
- The straight line is will determine if we determine two points on it.
- The plane in space is determined by one of the following cases :



Two parallel straight lines.



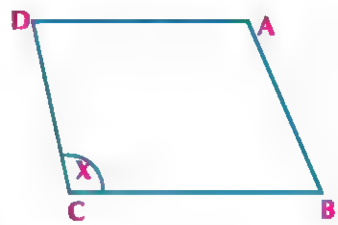
Two intersecting straight lines



A straight line and a point outside it

Three distinct and non - collinear points

- For any point in the space, there are an infinite number of planes passing through it.

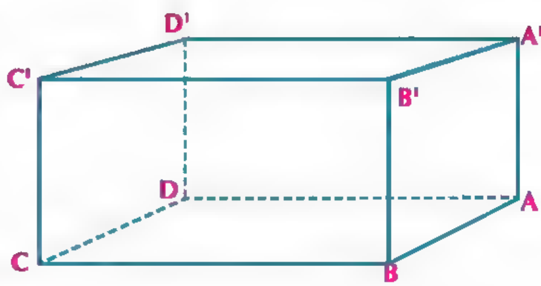


Plane: is a surface with no ends such that any straight line passes through two points on it lies completely on this surface . in the given figure :the plane is denoted by the symbol X .you may use any other symbol as Y , Z , …….or you can denoted it by three letters as ABC , …… ,the plane has no end from its sides and it may represented by a triangle ,a square ,a rectangle ,a parallelogram or a circle ,….. .

Space: is an infinite number of points contains all figures ,planes and solids we will study.

Example

(1) Meditate the following figure ,and then answer the following questions



a) Write three straight lines passes through the point A.

b) Write the straight lines passing through both of the points A and B all together

c) Write three planes passes through the point A

d) Write three planes passing through both of the points A and B all together.

Solution

a) $\overleftrightarrow{AB}$, $\overleftrightarrow{AA'}$, $\overleftrightarrow{AD}$

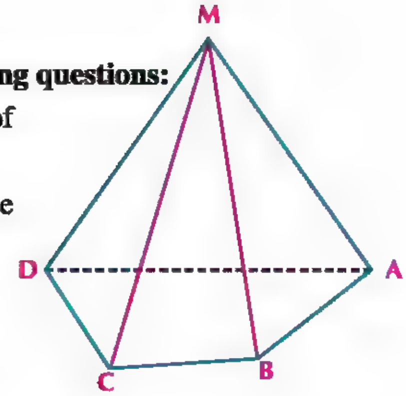
b) $\overleftrightarrow{AB}$

c) ABB', ABC, ADD'

d) ABB', ABC, ABC'D'

Try to solve

1 Meditate the following figure, and then answer the following questions:

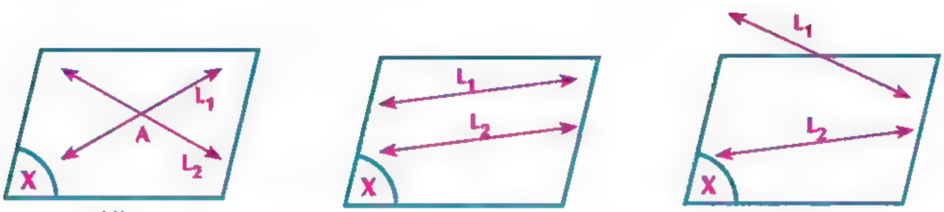


a How many straight lines in this figure? State the names of the straight lines passing through the point A.

b How many planes in this figure? State the names of three planes passing through the point A.

The relation between two straight lines in the space:

Meditate the following figures, and then complete:



(1) (2) (3)

1- The two intersected straight lines: are two straight lines lie in the same ....and having a common ....

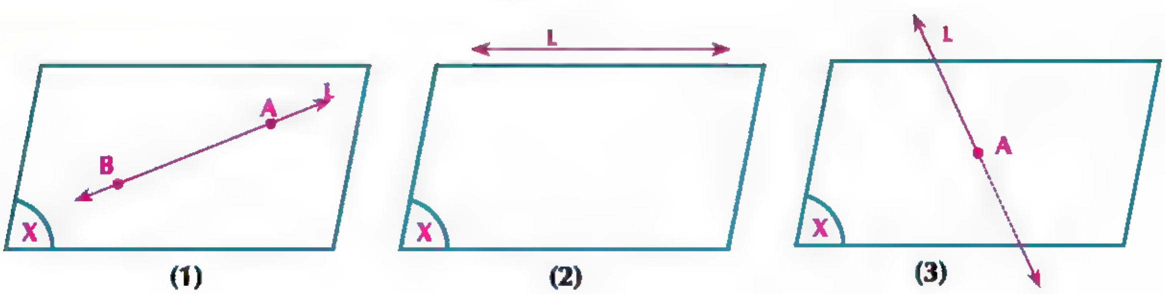
2- The two parallel straight lines: are two straight lines lie in the same ....and does not have a common ....

3- The two skew straight lines: are two straight lines does not contained in the same .... and they are not ....

Critical thinking: The two skew straight lines are neither parallel nor intersecting. Explain that.

The relation between a straight line and a plane in the space

Meditate the following figures, and then complete:

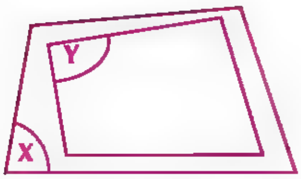


(1) (2) (3)

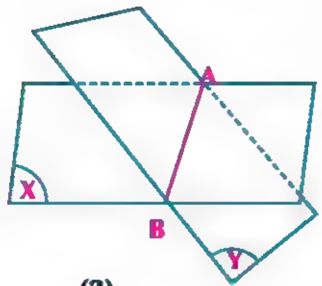
- The straight line is parallel to the plane in figure ....
- The straight line is intersected with the plane in figure ....
- The straight line contained in the plane in figure ....

The relation between two planes in the space

Meditate the following figures, and then complete:



(1)



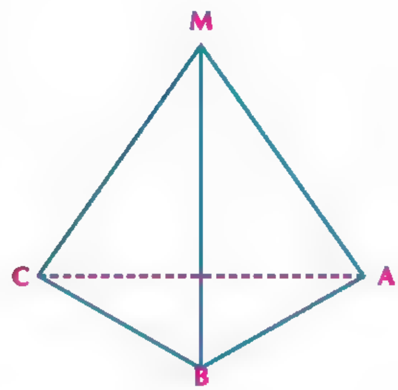
(2)

(3)

- The two planes are parallel in figure ……..
- The two planes are coincident in figure ………
- The two planes are intersected in figure ……….

Example

2 Meditate the following figure, and then complete:



a The plane MAB ∩ The plane MBC = ………

b The plane MBC ∩ The plane ABC = ………

c $\overleftrightarrow{MB}$ ∩ The plane ABC = ………

d $\overleftrightarrow{MC} \cap \overleftrightarrow{AB}$ = ………

e The plane MAB ∩ The plane MBC ∩ The plane MAC = ………

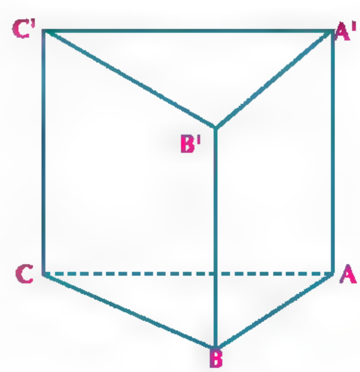
Solution

a $\overleftrightarrow{MB}$ b $\overleftrightarrow{BC}$ c {B}

d ϕ (because they are two skew straight lines) e {M}

Try to solve

2 Meditate the following figure, and then complete::



a The plane ABB'A' ∩ The plane BCC'B' = ………

b The plane ABC ∩ The plane A'B'C' = ………

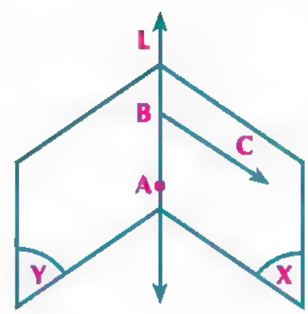
c $\overleftrightarrow{AC} \cap \overleftrightarrow{A'C'}$ = ………

d $\overleftrightarrow{BB'}$ ∩ The plane ABC = ………

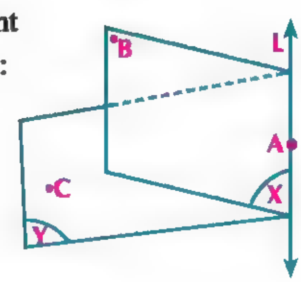
Exercise (3-1)

Complete the following:

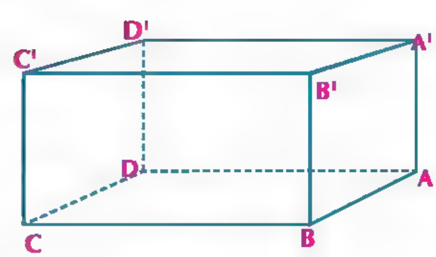
1. If the straight line L // the plane X, then $L \cap X =$ ....
2. If the straight line $L \subset$ the plane X, then $L \cap X =$ ....
3. If the straight line L_1 // the straight line L_2 ,then $L_1 \cap L_2 =$ ....
4. If X and Y are two planes such that: $X \cap Y = \phi$, then X.......... Y
5. The two skew straight lines are neither .... nor ....
6. **State the number of planes that passes through the following:**
   - **a** One given point
   - **b** Two different points
   - **c** Three collinear points
   - **d** Three non-collinear points.
   - **e** Four non-coplanar points



7. Meditate the following figure, and then complete using one of the following symbols ($\in$, $\notin$, $\subset$, $\not\subset$)
   - **a** L X
   - **b** A X
   - **c** C .... Y
   - **d** $\overleftrightarrow{BC}$ .... Y



8. In the opposite figure : X,Y are two planes intersected at the straight line L ,$A \in L$,$B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$ Complete the following:
   - **a** The plane X $\cap$ The plane A B C = ...
   - **b** The plane Y $\cap$ The plane A B C = ...
   - **c** The plane X $\cap$ The plane Y $\cap$ The plane A B C = ...
9. **Meditate the following figure, and then complete the following:**
   - **a** The plane ABCD // The plane ....
   - **b** The plane BCC'B' // The plane ....
   - **c** The plane ABB'A' $\cap$ The plane ABCD = ...
   - **d** The plane ABB'A' $\cap$ The plane DCC'D'= ....
   - **e** The plane DCC'D' $\cap$ The plane A B C D $\cap$ The plane ADD'A'= ....



10. Put the sign (✓) for the correct answer and the sign (✗) for the incorrect answer where L_1, and L_2 are two straight lines and X,Y are two planes:

   a. If $L_1 \cap L_2 = \phi$ then $L_1 // L_2$ or L_1, L_2 are skew
   b. If $L_1 \cap X = \phi$ then $L_1 // X$
   c. If $L_2 \cap X = L_2$ then $L_2 \subset X$
   d. If $L_2 \subset Y$ then $L_2 \cap Y = \phi$
   e. If $X \cap Y = \phi$ then $X // Y$
   f. If X = Y then X , Y are coincident

Choose the correct answer for each of the following:

11. Any four non-coplanar points form :
   a. Two planes
   b. three planes
   c. four planes
   d. no plane

12. If two plane have two common points A and B, then they will be:
   a. Coincident
   b. intersected at $\overleftrightarrow{AB}$
   c. Intersected at a straight line parallel to $\overleftrightarrow{AB}$
   d. With a third common point does not belong to $\overleftrightarrow{AB}$

13. $\overleftrightarrow{AB}$ // the plane X if
   a. $\overline{AB} \cap X = \phi$
   b. A and B lie in two different sides from X
   c. A and B with two different distances from X
   d. $\overleftrightarrow{AB} \cap X = \phi$

14. The two straight lines L_1 and L_2 are parallel if:
   a. $L_1 \cap L_2 = \phi$
   b. $L_1 \cup L_2$ lie in the same plane
   c. $L_1 \cap L_2 = \phi$, L_1 , L_2 located in the same plane.
   d. $L_1 \cap L_2 = \phi$, L_1 , L_2 does not located in the same plane.

15. The two straight lines are skew, if they are:
   a. not parallel.
   b. not coincident.
   c. not located in the same plane.
   d. located in the same plane.

Critical thinking:

16. Represent by drawing that: if three planes are intersected in pairs, then their lines of intersection will be either parallel or met at a point.

3 - 2 Pyramid and Cone

We will learn

- Properties of some figures:
- Pyramid - Regular Pyramid - Right Pyramid - Cone - Right cone.
- The concept of net solid.
- Conclude properties of the solid from its net.
- Drawing the net of a solid .
- Modeling and solving the problems of mathematical life situations. Using the properties of pyramid and right cone.

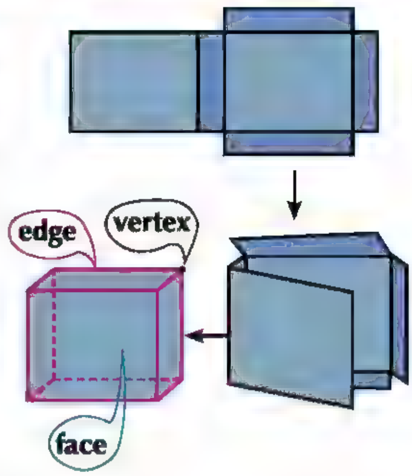
Key - term

- Pyramid - cone - height lateral face - lateral edge - slant height - Regular pyramid - Right pyramid - Net - Right circular cone.

Materials

- Geometrical instrument
- Scientific calculator.
- Drawing programs.

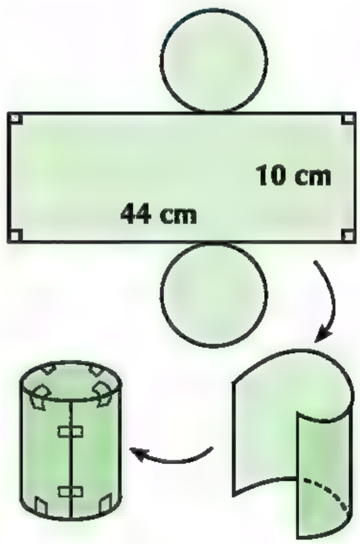
Many containers are manufactured by folding the flat cardboard to a three dimensional form to mobilize the factories, products before marketing so they occupy the vaccum of space, such as: cube and cuboid, ...



- How many faces and vertices does the cube have?
- How many edges does the cuboid have?
- Are all faces of the cube identical? Explain your Answer.

We call the figure which can be folded to form a solid by a net solid and from which we deduce the properties of the solid.

The opposite figure shows a right circular cylinder net, notice:

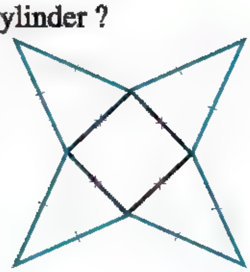


1 - The bases of the cylinder are identical and each of them has the shape of a circle.

2 - The lateral surface of the cylinder before its folding is a rectangle. whose dimensions are 44 cm, 10 cm then the height of the cylinder will be 10 cm.

What is the length of the radius of the base of the cylinder ?

Think:



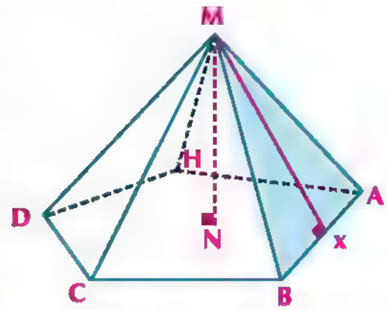
Do you know the name of a solid which will be configured by folding the opposite net? Deduce some of its properties?

Can you draw more than one net for the solid? Explain your answer.

Pyramid:

It's a solid of a single base and all of its other faces are triangles that have the same vertix the pyramid is called triangular, quadrelateral, pentagonal pyramid, according to the number of sides of its base.

Note: In the opposite figure MABCDH is a penta pyramid, with vertix M, and its base is the polygon ABCDH, its lateral faces are the surfaces of triangles MAB, MBC, MCD, MDH, MHA. and its lateral edges are $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$, $\overline{MH}$.



The height of the pyramid (MN) is the distance between the pyramid's vertix and its base level.

The slant height is the distance between the pyramid's vertix and any of its base edges.

Def.

Regular pyramid

It's a pyramid whose base is a regular polygon its centre is the foot of the perpendicular from the vertix to its base.

Properties of Regular pyramid

1 - Its lateral edges are equal in length.

2 - Its lateral faces are surfaces of isosceles congruent triangles.

3 - Slant heights are equal.

Note

Regular polygon is a polygon of sides that are equal in length and its angles are equal in measures. Its centre is a centre of drawn circle inside or outside it.

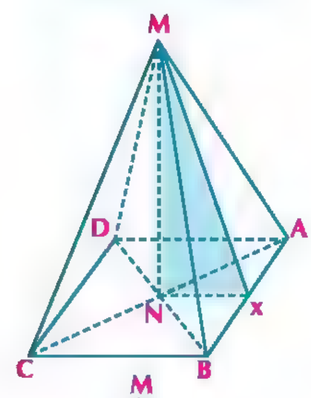
Note:

The perpendicular straight line drawn from the top of the pyramid to the level of its base is perpendicular to any straight line in it.

In the opposite figure: if $\overline{MN}$ is perpendicular to the base level

$\overline{MN} \perp \overline{AC}$, $\overline{MN} \perp \overline{BD}$, $\overline{MN} \perp \overline{NX}$

$\therefore$ The triangle MXN is a right angle triangle at N



Example

(1) MABCD is a regular quadrelateral pyramid. The length of its base side ABCD equals 10 cm and its hieght equals 12 cm. Find its slant height.

Solution:

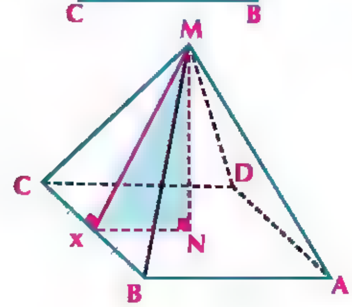
$\because$ The pyramid is regular quadrelateral

$\therefore$ $\overline{MN} \perp$ to the plane ABCD

where N is a point of intersection of the diagonals of the square ABCD, MN = 12 cm.

Let x be the midpoint of $\overline{BC}$ $\therefore$ $\overline{MX} \perp \overline{BC}$ (Why?)

and, $\overline{MX}$ is a slant height of the regular pyramid



In $\triangle$ DBC: N is the mid point of $\overline{DB}$, X is the midpoint of $\overline{BC}$

$\therefore NX = \frac{1}{2} DC = \frac{1}{2} \times 10 = 5$ cm

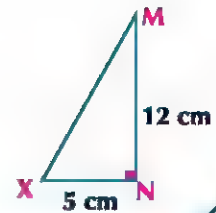
$\because \overline{MN} \perp$ plane ABCD

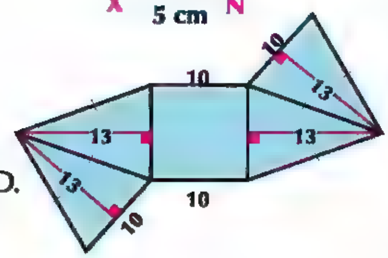
$\therefore \triangle$ MNX is a right angled at N

and, $(MX)^2 = (MN)^2 + (NX)^2 = (12)^2 + (5)^2 = 169$

$\therefore$ The slant height of the pyramid = 13 cm.

and the opposite figure shows one of the net of the pyramid MABCD.





Try to solve

1. MABCD is a regular quad. pyramid its height 20 cm, and its slant height 25 cm. Find the length of the side of the pyramid's base.

Right pyramid

The pyramid is right if and only if the projection of the perpendicular drawn from the vertex of the pyramid to its base passes through its geometrical centre.

Think:

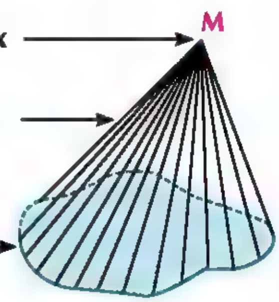
1 - Is the regular pyramid a right pyramid? Explain your answer.

2 - Are the slant height of the right pyramid equal in length?

Note: the pyramid is said to be triangular regular pyramid if all of its faces are equilateral triangles, so each face can represents a base of the pyramid.

Cone

It's a solid with a single base in the form of a closed curve with one vertix, its lateral surface consists of all points of line segments drawn from the vertix to the base of the curve, each of them is known as cone drawer.

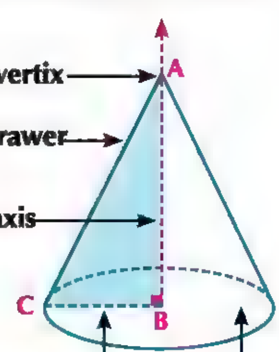


Right circular cone

It's a solid which is generated from the rotation of right angled triangle a compete revolution around one of the sides of the right angle as an axis.

Properties of a right circular cone:

The opposite figure shows a right circular cone constructed from rotation of right- angled triangle at B a complete rotation around $\overleftrightarrow{AB}$ as an axis. We will find that:



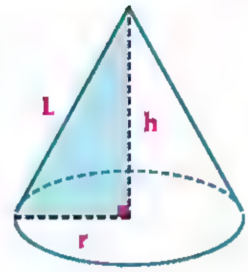
1- $\overline{AC}$ is a cone drawer, A is the vertex of the cone, the point C draw during rotation a circle its centre is the point B. The length of the radius of the circle equals to the length of $\overline{BC}$, the surface of the circle is the base of the cone.

2- $\overleftrightarrow{AB}$ the axis of the cone is perpendicular ($\perp$) to the base of the cone, the height of the cone equals to the length of $\overline{AB}$.

Example

(2) A right circular cone, the length of its drawer equals 17 cm, and its height equals 15 cm. Find the length of the radius of its circle.

Solution:



Let the length of the cone drawer = L, and its height = h,

the length of the radius of its base circle = r

$\because r^2 = L^2 - h^2$

$\therefore r^2 = (17)^2 - (15)^2 = 64$ $\qquad\qquad \therefore r = 8$ cm

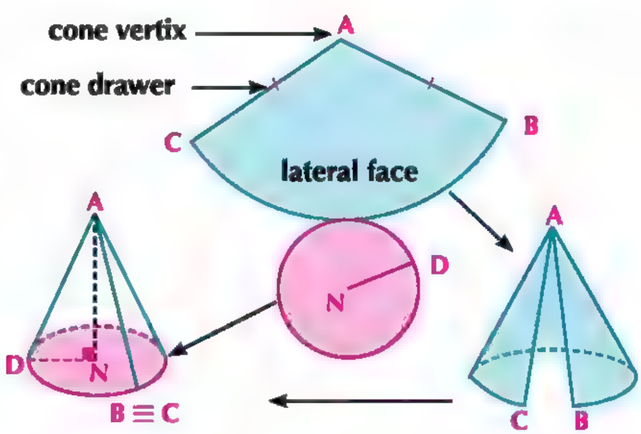
Try to solve

(2) Find in terms of π the circumference and the area of the base of a right circular cone. Its height equals 24 cm, and the length of its drawer equals 26 cm.

Think: A triangle ABC, AB = AC, D is the midpoint of $\overline{BC}$. If the triangle ABC rotates half a complete rotation around the axis $\overleftrightarrow{AD}$. Does a right circular cone arises? Explain your answer.

Right cone net:

The net of the right cone can be folded for forming cone-shaped containers as in the opposite figure, where:



1 - AB = AC = L (the length of the cone drawer)

2 - The circular sector ABC shows the lateral surface of the cone.

The length of $\overset{\frown}{BC} = 2\pi r$

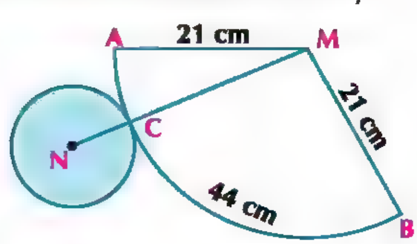
(r is the length of the radius of the cone's base)

3 - The height of the cone = the length of $\overline{AN}$

Example

(3) The opposite figure shows a net of a right cone, using the given data, find its height ($\pi \simeq \frac{22}{7}$).

Solution:



From the net of the cone we note that:

Length of the cone drawer = length of $\overline{MA}$ = 21 cm

Circumference of the cone base = length of $\overset{\frown}{AB}$ = 44 cm.

Length of the radius of the cone base = length of $\overline{CN}$ = r

When folding the net of the cone, the opposite figure arises then:

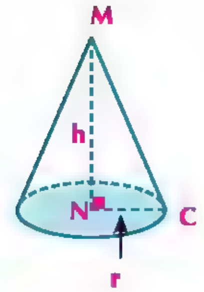
the height of the cone = the length of $\overline{MN} = h$

$\because 2\pi r = 44$ $\quad\therefore 2 \times \frac{22}{7} \times r = 44$ $\quad$ **then** $r = 7$ cm

$\because h^2 = L^2 - r^2$

$\therefore h^2 = (21)^2 - (7)^2 = 14 \times 28$ $\quad$ **then** $h = 14\sqrt{2}$ cm

$\therefore$ The height of the right circular cone $= 14\sqrt{2}$ cm.



Try to solve:

(3) In the previous net of the right cone, MA = 41cm, the length of $\overset{\frown}{AB} = 18\pi$ cm. Find the height of the cone.

Critical thinking: Is the following statement true:

the height of the right cone > the length of its drawer? (Explain your answer).

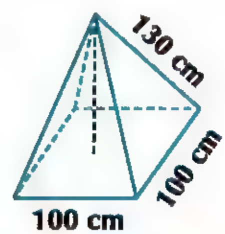
Exercises (3 - 2)

(1) In the regular pentagonal pyramid:

- **A** What the number of its lateral faces?
- **B** What the number of its faces?
- **C** What the number of its lateral edges?
- **D** What the number of its edges?
- **E** The pyramid has one vertix regardless of the vertices of the base. what is the number of all vertices of pentagonal pyramid? Is your answer prove Euler's rule for any solid, its base is a polygon. **"number of faces + number of vertices = number of edges +2"**

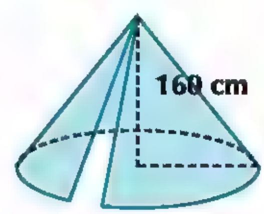
(2) In the regular pyramid, arrange the following lengths ascendingly

- **A** The length of the lateral edge
- **B** The height of the pyramid
- **C** The slant height



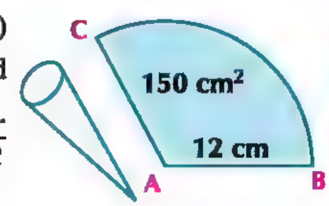
(3) **Civil geometry:** The opposite figure shows a water tank in the form of regular quadrangular pyramid. Using the given data, find each of the slant height and the height of the tank.

(4) **Connectivity with scouts:** A tent in the form of right circular cone, its height =160 cm and the circumference of its circular base = 753.6 cm. Find the length of the cone (tent) drawer.



(5) **Connecting to tourism:** The great pyramid of Giza (Khofo pyramid), the length of the side of its base is 232 m and its slant height is 186 m. Find the height of the pyramid.

(6) **Connecting to industry:** The frozen milk is encapsulated (Kept) on a right circular cone by folding a piece of healthy - insulated paper in the form of circular sector the length of its radius is 12 cm. and its area is 150 cm^2, where the two radii of the circle $\overline{AB}$, $\overline{AC}$ become in contact. Find the height of the cone.



[Note: The area of the sector $= \frac{1}{2}$ length of its arc × length radius of the circle].

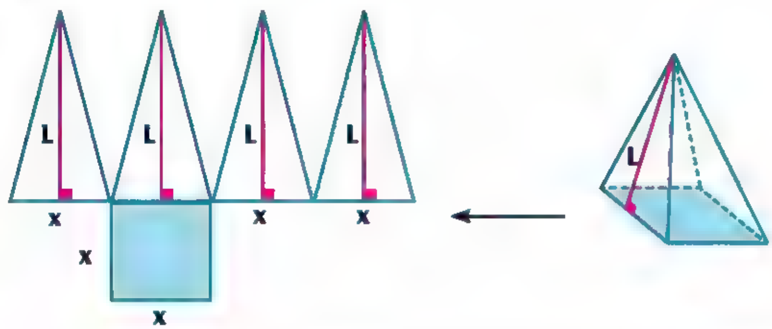
Total area of pyramids and cones

3 - 3

You have already learned the properties of the pyramid and the right circular cone. You have concluded some of them by studying their net. Can you calculate the lateral area and the total surface area of the regular pyramid and the right circular cone from their net? Explain.

Total area of a regular pyramid

The opposite figure shows a regular quadrelateral pyramid and one of its nets.



Note: The lateral faces are congruent isosceles triangles whose slants are with congruent heights (equal in length) and each of them = L.
The base of the pyramid is a regular polygon whose side length = X, then:

The lateral area of the pyramid

= sum of the area of the lateral faces.

$= \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L + \frac{1}{2}\ x \times L$

$= \frac{1}{2}\ (x + x + x + x)\ L$

$= \frac{1}{2}$ premiter the base of the pyramid × slant height.

Total area of the pyramid = lateral area of it + the area of its base.

Learn

The lateral area of the regular pyramid = $\frac{1}{2}$ the premiter of its base × its slant height.
The total area of the pyramid = its lateral area + its base area.

We will learn

- Finding the lateral area and the total surface area of the regular pyramid and right cone.
- Modeling and solving mathematical and life problems include the surface area of the pyramid and the right cone.

Key - term

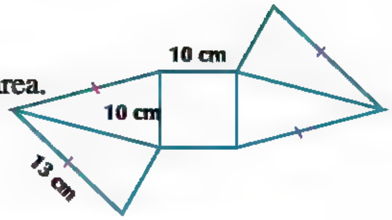
- Lateral surface area (L.S.A)
- Total surface area (T.S.A)

Materials

- *Scientific calculator*
- *Computer- Graphic program.*

Example

1. Using the opposite net, describe the solid and find its total area.



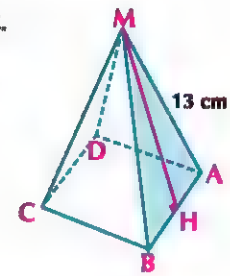
Solution

The net for a regular quadrangular pyramid.

Its base is square whose side length = 10 cm, the length of its lateral edge 13 cm.

$\because$ The lateral face MAB is an isosceles triangle, $\overline{MH}$ is a slant height.

$\therefore$ H is the mid point of $\overline{AB}$,which means AH = 5 cm



In $\triangle$ MHA which is right angled at H, we find that:

$(MH)^2 = (AM)^2 - (AH)^2$

$(MH)^2 = (13)^2 - (5)^2 = 144$

$\therefore$ M H = 12 cm

$\because$ The lateral area of the regular pyramid = $\frac{1}{2}$ Perimeter of the base $\times$ the slant height

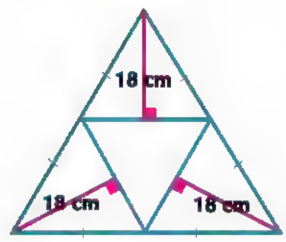
$\therefore$ The lateral area = $\frac{1}{2} \times (10 \times 4) \times 12 = 240$ cm^2

$\because$ The area of the pyramid base = $(10)^2 = 100$ cm^2

$\therefore$ The total area of the pyramid = 240 + 100 = 340 cm^2

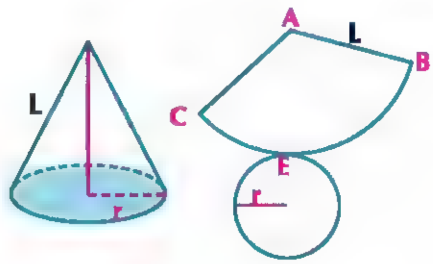
Try to solve

1. Using the opposite net, describe the solid and find its total area.



Total area of the right cone

From the right cone net. In the opposite figure:



The area of the sector ABC = $\frac{1}{2}$ AB $\times$ length of $\overset{\frown}{BC}$

$= \frac{1}{2}$ L $\times$ perimeter of cone base

$= \frac{1}{2}$ L $\times$ 2 πr = πLr

= Lateral area of the right cone

The total area of the right cone = its lateral surface area + area of its base

Learn

Lateral area of the right cone = πLr

Total area of the right cone = $\pi Lr + \pi r^2 = \pi r (L + r)$

Where L is the length of slant height, r is the radius of the circle.

Remember that

$\theta^{rad} = \frac{L}{r}$

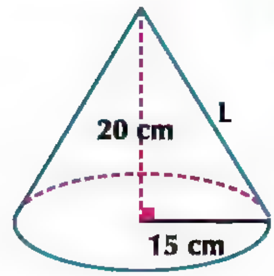
Perimeter of the circular sector = 2r + L

Area of the circular sector $= \frac{1}{2} Lr = \frac{1}{2} \theta^{rad} r^2$

Example

(2) Find the lateral area of a right cone, of base radius 15 cm, and its height 20 cm.

Solution



To find the length of cone drawer L

$\because$ $L^2 = (20)^2 + (15)^2 = 625$

$\therefore$ L $= 25$ cm

$\because$ Lateral area of right cone $= \pi Lr$, $r = 15$ cm

$\therefore$ Lateral area of right cone $= 25 \times 15\pi = 375\,\pi$ cm^2

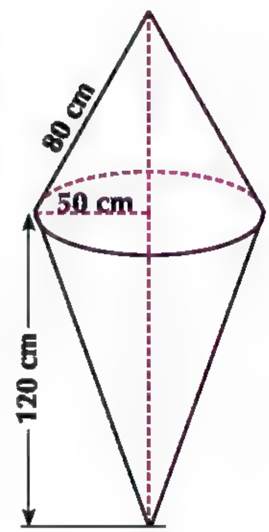
Try to solve

(2) Find the total area of a right cone if the length of its drawer = 17 cm and its height = 15 cm.

Example

(3) **Marine navigation:** The opposite figure shows guide sign (Shamandora) (Buoy) to determine the waterway, and it is in the form of two right cones have a common base.

Find the costs of its painting with a material which resists erosion factor, Note that each square meter of it costs 300 pound



Solution:

The area of the guide sign surface =

lateral area of the 1st cone + lateral area of the 2nd cone

First cone: $L_1 = 80$ cm , $r_1 = 50$ cm

$\therefore$ Lateral area $= 50 \times 80\,\pi = 4000\pi$ cm^2

Second cone: $h = 120$ cm , $r_2 = 50$ cm

$\therefore L_2 = \sqrt{(120)^2 + (50)^2} = 130$ cm

$\therefore$ Lateral area $= 50 \times 130\pi = 6500\,\pi$ cm^2

The area of the guide sign surface $= (4000 + 6500)\pi = 10500\,\pi$ cm^2

$\simeq 3.299$ squared meter

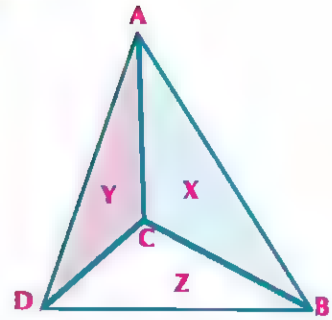
Cost of painting $= 3.299 \times 300 = 989.7$ pound

Try to solve

(3) Lamb cover is in the form of a right cone. The circumference of its base circle = 88 cm, its height = 20 cm. Calculate its area to the nearest square centimeter.

Exercises (3 -3)

1 The opposite figure represents a triangular pyramid, X , Y and Z are three planes. Complete the following



A $X \cap Y =$ .... **B** $X \cap Z =$ ....

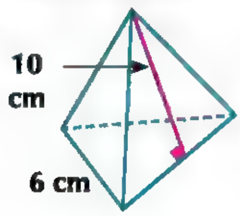
C $Y \cap Z =$ .... **D** $\overleftrightarrow{AB} \cap X =$ ....

E $\overleftrightarrow{BC} \cap$ .... X , $\overleftrightarrow{BC}$ .... Z

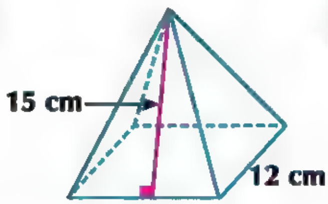
F $X \cap Y \cap Z =$ ....

2 Find lateral area, and total area for each regular pyramid, according to the given data

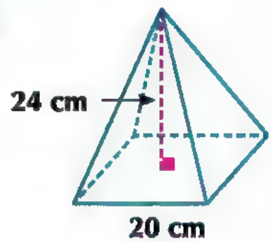
A



B

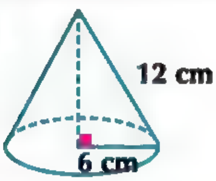


C

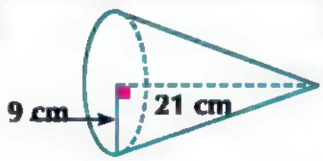


3 Find lateral area, and total area for each right cone, according to the given data.

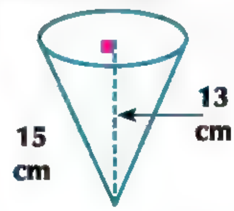
A



B



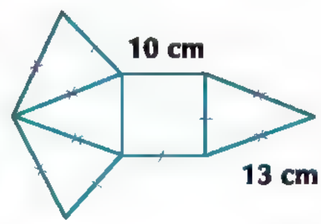
C



4 Hexagonal regular pyramid, the length of its base side 12 cm and its slant height $10\sqrt{3}$ cm. Find:

A Its lateral area **B** Its total area

5 **Connecting to industry:** products containers of a factory manufactured from cardboard by folding the net of the opposite figure.



A Find the area of the used cardboard to produce 1000 container.

B Calculate the costs of the used cardboard if each square meter costs 15 pound.

6 A piece of cardboard is folded in the form of circular sector the length of its radius 36 cm and the measure of its angle 210° for making a right circular cone has the greatest area. Find the height of the cone.

(The area of the sector $= \frac{1}{2} r^2 \theta^{rad}$, r radius of circular sector, θ^{rad} is the measure of the angle in radian)

7 Find the length of the radius of a right cone, if the length of the cone drawer 15 cm and its total area $154\,\pi$ cm^2.

Volumes of pyramids and cones

3 - 4

We will learn

- Finding the volume of regular pyramid.
- Finding the volume of right cone.
- Modeling and solving mathematical and life problems which include the volume of each of the regular pyramid and right cone.

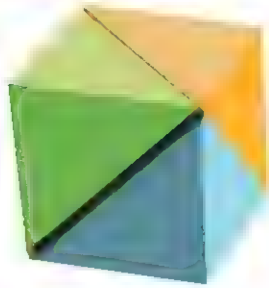
Key - term

- Vertex
- Base
- Face
- Axis
- Radius
- Volume

Materials

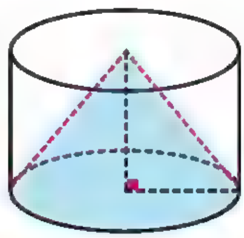
- Scientific calculator
- Computer- Graphic program.

Think and discuss



You have already learned how to calculate the volume of a right prism and the volume of a right circular cylinder.

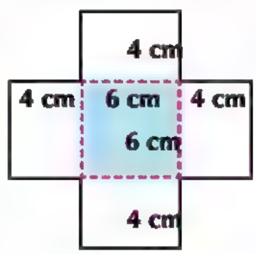
Can you estimate the volume of the pyramid in terms of the volume of the right prism which has the same base area and the same height?



Can you estimate the volume of the a right cone in terms of the volume of a cylinder which has the same base area and the same height?

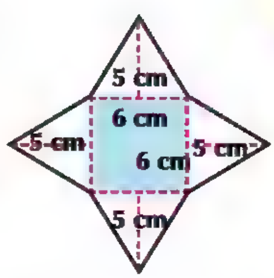
Activity

Comparison between the volumes of a pyramid and a prism which have the same area of the base and the same height.



1- Draw on a cardboard the net of the pyramid and prism which are shown in the opposite figure.

2- Cut and fold each net to make two models. One of them is the lateral surface of a quadrangular pyramid, while the other is a right prism, opened from the top.



3- Fill the pyramid with grains of rice or sand and empty it in the prism. Repeat this until the prism is completely filled.

Note that: the contents (grains of rice or sand) that you need to fill the prism, will completely fill three pyramids which means:

The volume of the pyramid = $\frac{1}{3}$ the volume of the prism which have the same base area of the pyramid (b), and the same height of the pyramid (h).

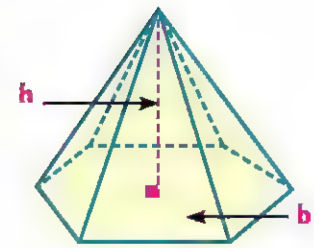
Volume of a Pyramid

Learn

The volume of the pyramid equals one third of the product of the area of its base multiplied by its height.

Which means: the volume of the pyramid = $\frac{1}{3}$ b × h

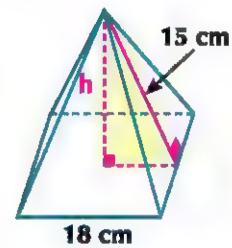
Where: (b) is the area of the base, (h) is the height of the pyramid.



Example

(1) Calculate the volume of regular quadrangular pyramids, the length of its base side is 18 cm and its slant height is 15 cm.

Solution:



First: calculation of the area of the pyramid's base (b)

$\because$ The pyramid is regular quadrangular.

$\therefore$ Its base is a square shape.

The area of the pyramid's base (b) = 18 × 18 = 324 cm^2

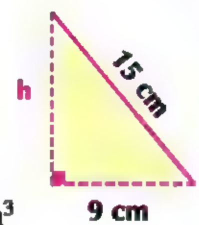
Second: calculation of the pyramid's height (h)

$\because h^2 + (9)^2 = (15)^2$ Pythagouras

$\therefore h^2 = (15)^2 - (9)^2 = 144$, $h = 12$ cm

$\because$ pyramid's volume = $\frac{1}{3}$b × h

$\therefore$ pyramid's volume = $\frac{1}{3} \times 324 \times 12 = 1296$ cm^3



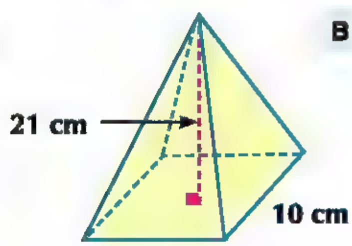
Remember that

surface area of a regular polygon of n side, length of each side is x equals $\frac{n}{4} x^2 \cot \frac{\pi}{n}$

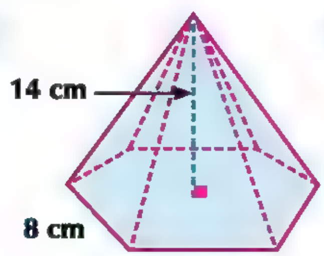
Try to solve

(1) Find the volume of a regular pyramid which is shown in each figure using the given date.

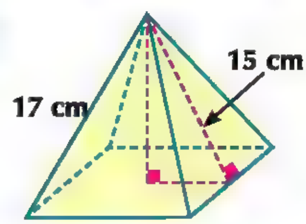
A



B



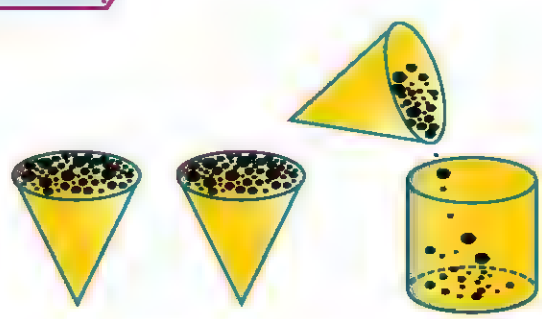
C



Think: When comparing the volume of right circular cone and right cylinder having the same base area and the same height, we find that:

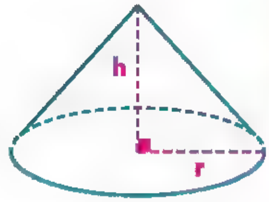
Volume of a cone = $\frac{1}{3}$ volume of cylinder.

How can you explain that mathematically?



Volume of a cone

Learn



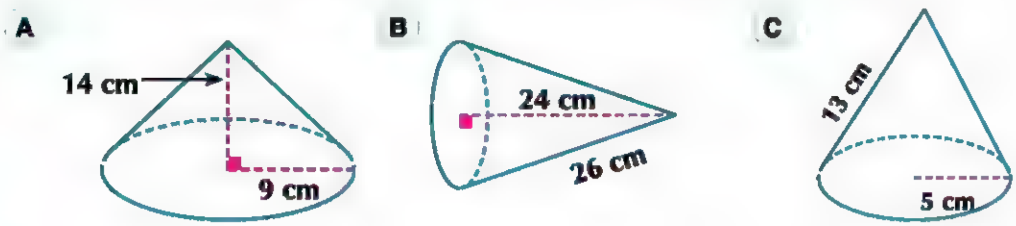
Volume of a cone equals one third product of the area of its base by its height.

which means: **The volume of the cone = $\frac{1}{3}\pi r^2 h$**

where (r) is the length of the radius of the cone's circle, (h) is the height of the cone.

Try to solve

(2) Find the volume of the right cone shown in the figure using the given data.



Example

(2) **Connecting to physics :** An alloy of pure gold in the form of a right cone. Its height is 4.2 cm and the length of its circle's radius is 1.5 cm. Find the density of the gold if the mass of the alloy is 191 gm.

Solution

$\therefore$ The volume of cone $= \frac{1}{3}\pi r^2 h$, $r = 1.5$ cm , $h = 4.2$ cm

$\therefore$ The volume of gold in alloy $= \frac{\pi}{3}(1.5)^2(4.2) = 9.896$ cm^3

$\therefore$ density $= \frac{\text{mass}}{\text{volume}}$ $\therefore$ density of gold $= \frac{191}{9.896} \simeq 19.3$ gm/cm^3

Try to solve

(3) A piece of chocolate in the form of a right cone. Its volume is 27π cm^3, the perimeter of its base 6π cm. Find its height.

Example

(3) **Connecting to Industry:** A pentagonal regular pyramid from copper, the length of its base polygon side is 10 cm, and its height is 42 cm. It's melted and converted to a right circular cone. The length of its base radius is 15 cm. If it had known that 10% of copper had been lost during melting and converting it, find the height of the cone to the nearest one decimal number.

Solution

$\because$ The area of the regular pentagon $= \frac{5}{4}x^2 \cot\frac{\pi}{5}$ (x is the length of its side)

$\therefore$ The area of the pyramid base $= \frac{5}{4} \times 10 \times 10\cot 36^\circ = \frac{125}{\tan 36^\circ} \simeq 172$ cm^2

$\because$ The volume of the pyramid $= \frac{1}{3}$ area of the base $\times$ the height $= \frac{172}{3} \times 42 = 2408\ cm^3$

$\therefore$ The volume of copper in the cone $= \frac{90}{100} \times 2408 = 2167.2\ cm^3$

$\frac{\pi}{3}(15)^2 h = 2167.2$ where h is the height of right cone

$\therefore h = \frac{2167.2 \times 3}{225\pi} \simeq 9.2$ cm

Try to solve

(4) A cube of wax, the length of its edge is 20 cm. It's melted and converted to a right circular cone, its height is 21 cm. Find the length of the base radius of the cone, if it is known that 12% of wax had been lost during melting and reforming.

Important note: Container capacity is estimated by the volume of the liquid which it contains. To calculate the capacity, the same laws of calculating the volumes are used. The measuring unit of capacity is litre.

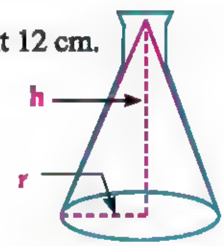
Remember that

Capacity being used for how much a container can hold.

1 litre = 1000 milliliter = 1000 cm^3 = dm^3

Example

(4) **Connecting to chemistry:** A conical flask, its capacity 154 ml, its height 12 cm. Find the length of its base radius ($\pi \simeq \frac{22}{7}$)



Solution

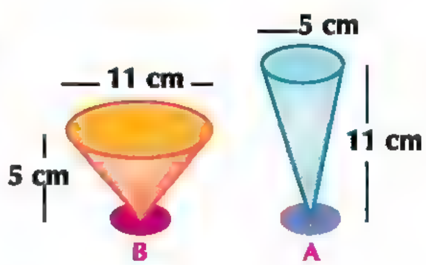
capacity of the flask = volume of right cone = 154 cm^3

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 12 = 154 \quad \therefore r^2 = \frac{49}{4}$

$\therefore r = 3.5$ cm

Try to solve

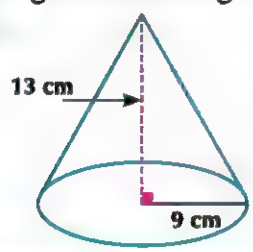
(5) A, B are two cups for drinking. which of them has greater capacity? Find the difference between their capacities.



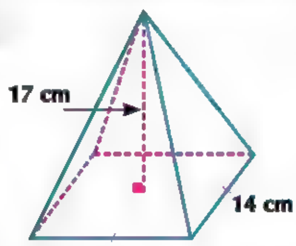
Exercises (3 - 4)

1. Find the volume of a regular quadrangular pyramid. The length of the side of its base =20 cm, and its height =36 cm.
2. Calculate to the nearest one decimal place, the volume of a regular pentagonal pyramid the length of its base side is 40 cm, and its height is 10 cm.
3. A regular quadrangular pyramid, its height 9 cm, and its volume 300 cm^3. Find the length of the side of its base polygon.
4. A regular quadrangular pyramid, the area of its base 700 cm^2, and its slant height 20 cm. Find its volume.
5. Which is greater in volume? A right cone the length of its base radius is 15 cm, and its height is 20 cm, or a regular quadrangular pyramid its height is 40 cm, and the perimeter of its base is 48 cm.
6. Find the volume of a right cone its base perimeter is 44 cm, and its height is 25 cm.
7. Find the volume of a right cone its lateral area is 220 cm^2, and the length of its drawer is 14 cm.
8. Arrange the following figures from the smallest volume to the largest volume.

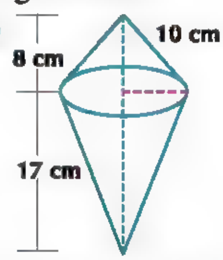
A



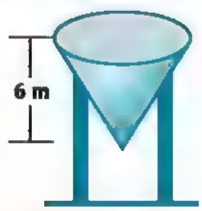
B



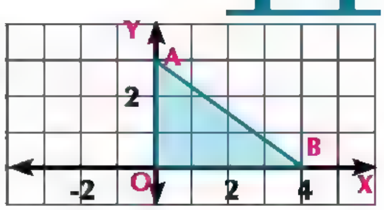
C



9. **Connecting to tourism :** A model of the great pyramid is made of metallic alloy its density is 3.2 gm/cm^3. If the length of the model side base 11.5 cm, and its height 7 cm, then calculate its mass to the nearest one decimal place.
10. **Connecting to physics:** A cylindrical shaped vessel contain water, A metal body in the form of a right cone, its height is 12cm, and the length of its base radius is 2 cm and is completely immersed in it Raising the surface of the water in the vessel with the value 1 cm. Find the length of base diameter of the vessel.



11. **Civil engineering:** A tank of water in the form of right cone, its volume is 32 πm^3 and its height is 6 m. Find the length of its base radius and its total area.
12. The opposite figure shows a coordinate perpendicular plane. calculate in terms of π the volume of solid generated when revolving triangle ABO one complete revolution around:



   A The x -axis **B** The y - axis.
13. **Critical thinking:** A right circular cone its volume is 100 cm^3.
   Find its volume when:
   A Its height is doubled **B** The length of its radius is doubled.
   C Its height is doubled and the length of its radius is doubled.
   What you conclude? Explain your answer.

3 - 5

Equation of a circle

We will learn

- Writing the equation of the circle in terms of its coordinate centre and the length of its radius.
- The general equation of the circle.
- determining the coordinate of the circle centre, and the length of its radius from the general equation of the circle.
- Modeling and solving life problems including the equation of the circle.

Key - term

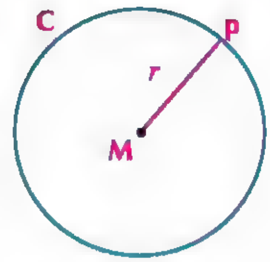
- Circle
- Center
- Radius
- Diameter
- Cartesian plane
- Equation
- General Form

Materials

- Scientific calculator
- Graph paper

Introduction :

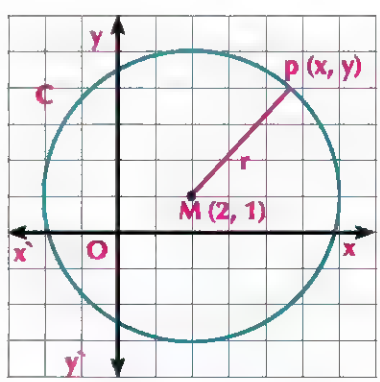
You know that the circle is a set of plane points which are equidistant from a fixed point in the plane.



The fixed point is called the "center", and its is usually denoted by the symbol M, and the distance from the centre to any point on the circle is called "radius". It is denoted by the symbol r.

The equation of a circle:

Equation of a circle is a relation between the x-coordinate and the y-coordinate for any point belongs to this circle. Each ordered pair (x, y) satisfies this relation (equation) and represents a point belonging to this circle.



In the perpendicular coordinate plane, if the point P (x, y) belongs to the circle c.

The length of its radius equals 4 units, and its centre is the point M (2.1), then

MP = r = 4, by applying the law of distance between two points, then:

$(x - 2)^2 + (y - 1)^2 = (4)^2$

$\therefore (x - 2)^2 + (y - 1)^2 = 16$

which is the equation of the circle c

Remember that

The distance between the two points (x_1, y_1), (x_2, y_2)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

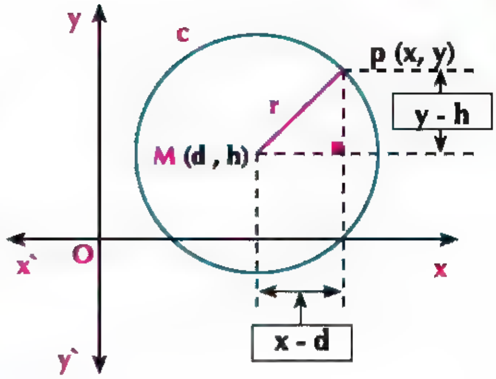
Learn

The equation of a circle

(In terms of the coordinates of its centre and length of its radius)

In a perpendicular coordinates plane:

If the point P (x , y) which belongs to the circle C whose centre is the point M (d, h) and length of its radius equal r , then the equation of the circle is:



$$(x - d)^2 + (y - h)^2 = r^2$$

Example

1) Write the equation of a circle C, its centre is the point M (5 , 2) and the length of its radius equals 6 units.

Solution

Let the point P (x , y) $\in$ the circle C

$\because$ The center of the circle is M (5 , 2), the length of the radius 6 units

$\therefore d = 5 \ , \ h = 2 \ , \ r = 6$

, then the equation of the circle is: $(x - 5)^2 + (y - 2)^2 = (6)^2$

then: $(x - 5)^2 + (y - 2)^2 = 36$

Try to solve

1) Write the equation of a circle if its centre:

A M (4 , -3) , the length of its radius equals 5 units.

B M (7 , -1) , the length of its diameter equals 8 units.

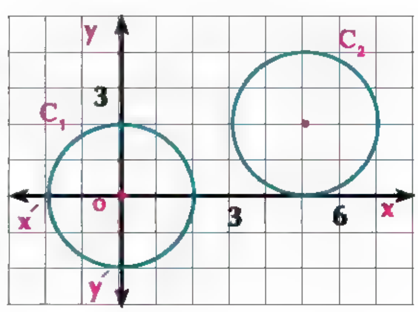
C M (2 , 0) , the length of its diameter equals $\sqrt{28}$ units.

D M (0 , -5) , and passes through the point A(-2 , -9)

E The origin point, the length of its radius equals r units.

Example

2) The opposite figure shows the two circles C_1,C_2. Prove that the two circles are congruent, then find the equation of each of them.



Solution

The two circles are congruent if the two radii have the same length.

The circle C_1: its centre (0 . 0), the length of its radius $r_1 = 2$ units.

The circle C_2: its centre (5 . 2), the length of its radius $r_2 = 2$ units.

$\because r_1 = r_2 = 2$ $\therefore$ The two circles are congruent.

then: the equation of the circle c_1: $x^2 + y^2 = 4$,

the equation of the circle c_2: $(x - 5)^2 + (y - 2)^2 = 4$

Note: The circle C_2 is the image of the circle C_1, by a translation (5 , 2)

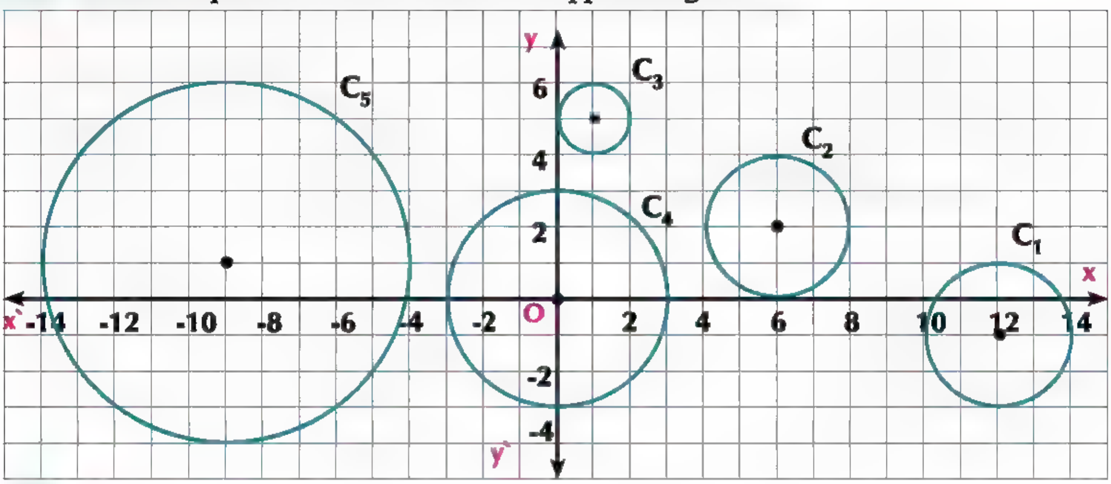
Remember that

The image of the point (h, k) by translation (a, b) is (h+a, k+b)

Critical thinking: If the circle C_3 is the image of the circle C_1 by a translation (-4 , 3), then write the equation of the circle C_3.

Try to solve

(2) **A** Write the equation of each circle in the opposite figure:



B Which of the previous circles are congruent? Explain your answer.

Think: what is the position of the point (x_1, y_1) with respect to the circle C: $(x - d)^2 + (y - h)^2 = r^2$ if:

A $(x_1 - d)^2 + (y_1 - h)^2 > r^2$ **B** $(x_1 - d)^2 + (y_1 - h)^2 < r^2$

Example

(3) Show that the point (4 , -1) is a point on the circle C its equation: $(x - 3)^2 + (y - 5)^2 = 37$

Solution

By substituting the coordinates of the point (4 , -1) in the right hand side of the equation of the circle.

$\therefore$ $(4 - 3)^2 + (-1 - 5)^2 = 1 + 36 = 37$ = left hand side

$\therefore$ the point (4 , -1) belongs to the circle C.

Note that: If $(x_1 - 3)^2 + (y_1 - 5)^2 > 37$ then the point (x_1 , y_1) lies outside the circle C.

and if $(x_1 - 3)^2 + (y_1 - 5)^2 < 37$ then the point (x_1 , y_1) lies inside the circle C.

Try to solve

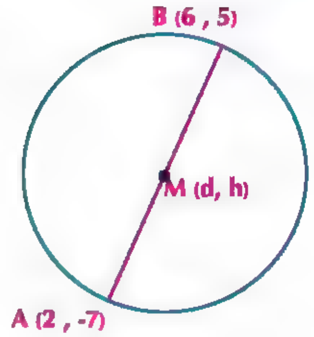
(3) Show which of the following points belongs to the circle C, whose equation: $(x - 6)^2 + (y + 1)^2 = 25$, then determine the position of other points with respect to the circle C. where:

A(9 , 3) , B(7 , 5) , C(3 , 3) , E(2 , -3)

Example

(4) Write the equation of the circle whose diameter is $\overline{BA}$ where A(2 , -7) , B(6 , 5)

Solution



Let the point M (d , h) be the center of the circle whose diameter is $\overline{BA}$, so the point M will be the midpoint of $\overline{BA}$.

$\therefore$ The coordinates of M: $d = \frac{2+6}{2} = 4$, $h = \frac{-7+5}{2} = -1$

$r^2 = (AM)^2 = (4-2)^2 + [-1-(-7)]^2$

$= (2)^2 + (6)^2 = 40$

The equation of the circle will be: $(x - 4)^2 + [y - (-1)]^2 = 40$

Which means: $(x - 4)^2 + (y + 1)^2 = 40$

Think: Is the point (6 , 5) satisfy the equation of the circle? Why?

Is the point (6 , -7) belongs to the previous circle, Explain your answer.

Try to solve

(4) Write the equation of the circle if:

A Its centre is the point M (-2 , 7), and passes through the point A(2 , 10).

B Its centre is the point M (5 , 4), and touches the straight line $x = 2$

C Its centre M lies in the 1st quad. of the coordinates plane where its radius is of length 3 units, and the two straight lines $x = 1$, $y = 2$ are two tangents to it.

Example

(5) Find the coordinates of the center, and length of its radius in each of the following circle:

A $(x-2)^2 + (y+3)^2 = 17$ **B** $(x+1)^2 + y^2 = 16$

Solution

We know that the equation of a circle in terms of coordinates of the centre (d , h), and the length of its radius r is :

$(x-d)^2 + (y-h)^2 = r^2$

Compare each of the algebraic expressions in the equation by its corresponding in the given equation. We find:

A $x - d = x - 2$ $\therefore d = 2$

$y - h = y + 3$ $\therefore h = -3$

$r^2 = 17$ $\therefore r = \sqrt{17}$

then the centre of the circle will be the point (2 , -3) and the length of its radius equals $\sqrt{17}$ units.

B $x - d = x + 1$ $\therefore d = -1$

$y - h = y$ $\therefore h = 0$

$r^2 = 16$ $\therefore r = 4$

$\therefore$ The centre of the circle is the point (-1 , 0) and the length of its radius equals 4 units.

Try to solve

5. Which of the given circles represents a circle whose centre is (3 , -4) and the length of its radius equals 3 units.

A $(x - 3)^2 + (y - 4)^2 = 9$ **B** $(x + 3)^2 + (y - 4)^2 = 9$

C $(x - 3)^2 + (y + 4)^2 = 9$ **D** $(x + 3)^2 + (y + 4)^2 = 9$

6. Find the coordinates of the center and length of radius of each of the following circles:-

A $(x - 3)^2 + (y + 5)^2 = 15$ **B** $x^2 + (y + 4)^2 = 9$

C $(x + 1)^2 + (y + 7)^2 = \frac{3}{4}$ **D** $(x + 1)^2 = 13 - y^2$

Learn

General form of the equation of a circle

You know that the equation of the circle whose centre (d , h), and the length of its radius equals r units:

is : $(x - d)^2 + (y - h)^2 = r^2$ by simpliying the expression

$\therefore x^2 + y^2 - 2dx - 2hy + d^2 + h^2 - r^2 = \text{zero}$ (1)

$\because$ d , h , r constant $\therefore$ The expression $d^2 + h^2 - r^2 = C$ where C is a constant vlue By butting $L = -d$, $k = -h$, $C = d^2 + h^2 - r^2$

Then the equation will be in the form $\boxed{x^2 + y^2 + 2Lx + 2ky + C = 0}$

and it's called the general form of the equation of a circle whose centre (-L , -k), and the length of its radius equals r, where

$r = \sqrt{L^2 + k^2 - C}$, $L^2 + k^2 - C > 0$

Example

6. Find the general form of the equation of the circle whose centre (6 , -3) and the length of its radius equals 5 units.

Solution

$\because$ the centre of the circle in the general form of the equation of a circle is (-L , -K)

, the centre of the circle is (6 , -3) given

$\therefore L = -6$, $k = 3$

$\because r = 5$, $C = L^2 + k^2 - r^2$

$\therefore C = (-6)^2 + (3)^2 - (5)^2 = 20$

The general form of the equation of the circle is: $x^2 + y^2 - 12x + 6y + 20 = 0$.

The validity of the solution can be verified using the equation of the circle:
$(x - 6)^2 + (y + 3)^2 = 25$ then simplify and compare it to the results.

Try to solve

(7) Write the general form of the equation of the circle if:

A its centre is the point M (-2 , 5), and the length of its radius equals $\sqrt{57}$ units.

B its centre is the point N (5 , -3), and passes through the point B (2 , 1).

Example

(7) Write the general form of the equation of the circle if the two points A(4 , 2) , B(-1 , -3) are the end points of its diameter.

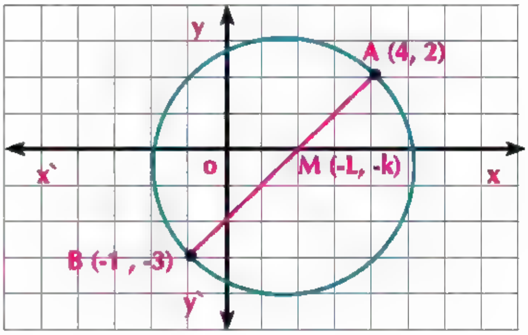
Solution

Let the point M (-L , -k) be the center of the circle whose diameter is $\overline{AB}$

$\therefore$ M is the midpoint of $\overline{AB}$, and the coordinates of the point M is $(\frac{4-1}{2}, \frac{2-3}{2})$

$\therefore -L = \frac{3}{2}$ $\qquad L = \frac{-3}{2}$

$-k = \frac{-1}{2}$ $\qquad k = \frac{1}{2}$



Substituting by L, k in the general form of the equation of the circle:

$x^2 + y^2 + 2Lx + 2ky + C = 0$

$\therefore x^2 + y^2 - 3x + y + C = 0$ $\qquad$ **(1)**

$\because$ The circle passes through the point A (4 , 2), so it verify its equation

$\therefore (4)^2 + (2)^2 - 3(4) + 2 + C = 0$ $\therefore C = -10$

By substituting in the equation **(1)**

$\therefore$ The general form of the equation of the circle is: $x^2 + y^2 - 3x + y - 10 = 0$

Try to solve

(8) If the points A (3 ,-2), B (3 , 8), C(-1 , 0) belong to the same circle. then prove that $\overline{AB}$ is a diameter in it, then write the general form of its equation.

Important note

From the general form of the equation of the circle : $x^2 + y^2 + 2Lx + 2Ky + C = 0$

We conclude that:

First: The equation is of the 2nd degree in x , y

Second: coefficient of x^2 = coefficient of y^2 = unity.

Third: Free form the term xy which means coefficient of xy = 0,

The second degree equation in x, y represents a circle, if the previous three conditions are verified, and $L^2 + K^2 - C > 0$.

Learn

Determine the coordinates of the centre of a circle and its radius

To determine the coordinates of the centre of the circle and the length of its radius from the general form of its equation:

1- Verify firstly to put the equation in the general form where:

the coefficient of x^2 = coefficient y^2 = unity

2- The coordinates of the centre is (-L , -k) $\therefore \left(\frac{-\text{ coefficient } x}{2}, \frac{-\text{ coefficient } y}{2}\right)$

3- The length of the radius of the circle equal r **where** $r = \sqrt{L^2 + k^2 - C}$, $L^2 + k^2 - C > 0$

Example

8 Which of the following equations represent a circle? And if it is a circle, find its centre and the length of its radius

A $3x^2 + 2y^2 + 6x - 8y - 10 = 0$

B $x^2 + y^2 + 4x + 25 = 0$

C $2x^2 + 2y^2 - 12x + 8y - 30 = 0$

D $4x^2 + 4y^2 = 49$

E $x^2 + y^2 + 2xy + 3 = 0$

Solution

A The coefficient of $x^2 \neq$ the coefficient of y^2 $\therefore$ the equation is not a circle

B The coefficient of x^2 = the coefficient of y^2 = unity, the equation is free of the term containing x y

$L = \frac{4}{2} = 2$, $k = \frac{0}{2} = 0$, $C = 25$

$\therefore L^2 + k^2 - C = (2)^2 + (0)^2 - 25 < 0$

$\therefore$ the equation is not a circle

C Divide both sides by 2 $\therefore x^2 + y^2 - 6x + 4y - 15 = 0$

$\therefore$ The coefficient of x^2 = the coefficient of y^2 = unity, the equation is free of the term containing xy

$L = -3$, $k = 2$, $C = -15$

$\therefore L^2 + k^2 - C = (-3)^2 + (2)^2 - (-15) = 28 > 0$

$\therefore$ The equation is for a circle its centre is (3 , -2) , $r = \sqrt{28} = 2\sqrt{7}$ units

D Divide both sides by 4 $\therefore x^2 + y^2 = \frac{49}{4}$

$\therefore$ The coefficient of x^2 = the coefficient of y^2 = unity , the equation is free of the term containing xy

$L = 0$, $k = 0$, $C = \frac{49}{4}$ $\therefore L^2 + k^2 - C = \frac{49}{4} > 0$

$\therefore$ The equation is a circle, its centre the origin point , $r = \sqrt{\frac{49}{4}} = \frac{7}{2}$ unit

E The equation contains the term xy $\therefore$ The equation is not a circle

Try to solve

9 Which of the following equations represent a circle? And if it is for a circle, find its centre and the length of its radius.

A $x^2 + y^2 - 6x + 4y + 17 = 0$

B $x^2 + y^2 + 4x - 2y = 0$

C $2x^2 + 2y^2 - 4x + 39 = 0$

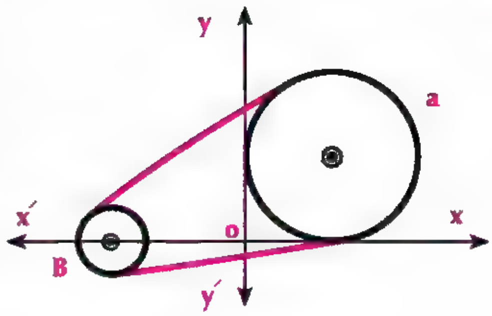
D $x^2 + y^2 - 2xy - 6 = 0$

Critical thinking: Are the two circles $C_1 : x^2 + y^2 - 10x - 8y + 16 = 0$

$C_2 : x^2 + y^2 + 14x + 10y - 26 = 0$ touch externally? Explain your answer.

Example

(9) **Connecting to industry :** The opposite figure shows a pulley A in a machine which touches the two coordinate axes, rotates by a wire which passes through a small pulley B, the equation of its circle: $x^2 + y^2 + 14x + 45 = 0$ Find:



A The equation of the circle of pulley A, given that the length of its radius equal 5 units.

B The distance between the centre of the pulleys, if each unit in the coordinate axes represent 6 cm.

Solution

A $\because$ The pulley A touches the axes of the coordinates, and the length of its radius equal 5 units.

$\therefore$ The centre of its circle is the point M (5 , 5) $\qquad \therefore L = -5 \;, \; k = -5$

$\because C = L^2 + k^2 - r^2 \qquad \therefore \; C = (-5)^2 + (-5)^2 - (5)^2 = 25$

and its equation will be: $x^2 + y^2 - 10x - 10y + 25 = 0$

B $\because$ The equation of the pulley circle B: $x^2 + y^2 + 14x + 45 = 0$

$\therefore L = 7 \qquad k = 0 \qquad C = 45 \qquad r = \sqrt{49 - 45} = 2$

and its centre will be the point N (-7 , 0) and the length of its radius equals 2 units

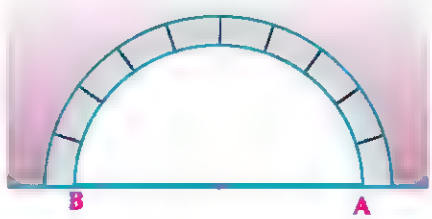
$\therefore$ The distance between the centers of two circles = M N= $\sqrt{(5 + 7)^2 + (5)^2} = 13$ units

$\because$ Every unit in the coordinates plane represents 6 cm

$\therefore$ The distance between the two pulleys $= 13 \times 6 = 78$ cm

Try to solve

(10) **Connecting with roads:** The opposite figure shows a vertical cross section in one of the circular tunnels passage of cars, the equation of its circle:
$x^2 + y^2 - 4x - 6y - 12 = 0$, $\overline{AB}$ is a diameter in it.
Find the maximum height of the tunnel, if the unit of the length of the coordinate plane represents 70 cm.

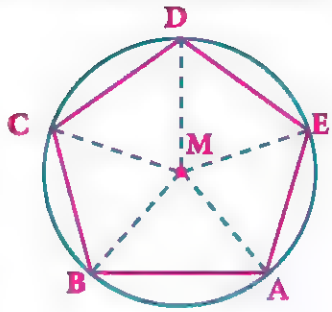


Example

(10) **Connecting with geometry:** Find to the nearest cm^2 the surface area of a regular pentagon:
If the circle: $x^2 + y^2 + 6x - 12y + 5 = 0$ passes through its vertices knowing that each unit of the coordinate plane represent 5 cm.

Solution

Let the point M is the centre of the circle which passes through the vertices of the regular pentagon ABCDE, then:



AB = BC = CD = DE = AE (chords in the circle M)

$\therefore$ m($\angle$AMB) = m($\angle$BMC) = ... = $\frac{360^\circ}{5}$ = 72°

and it is noticed that the figure ABCDE is divided into five congruent triangles.

Then the area of the regular pentagon = 5 × area of Δ MAB

$= 5 \times \frac{1}{2}$ MA × MB sin 72°

$= \frac{5}{2} r^2 \sin 72^\circ$ (1)

from the equation of the circle: L = 3 k = -6 C = 5

$\because r^2 = L^2 + k^2 - C$ $\therefore r^2 = 9 + 36 - 5 = 40$

and by substituting in (1)

$\therefore$ area of regular pentagon = $\frac{5}{2}$ (40) sin 72° = 95.10565 squared unit

$\because$ Each unit length in the coordinate plane represents 5 cm.

$\therefore$ Each squared unit length in the coordinate plane represents area = $(5)^2 = 25$ cm^2

the area of the regular pentagon = 95.10565 × 25 $\simeq$ 2378 cm^2

Remember that

The area of any regular polygon with n sides and r is the radius of its cicumcircle $\frac{n}{2} r^2 \sin \frac{360}{n}$

Exercises (3 - 5)

Choose the correct answer from those given:

1. The point (2 , 0) lies on the:
   - A x - axis
   - B y - axis
   - C straight line y= 2x
   - D circle $x^2 + y^2 = 9$
2. If A(3 , -7) , B(-3 , 5) , then the coordinates of the midpoint of $\overline{AB}$ is ....
   - A (0 , 1)
   - B (1 , 0)
   - C (0 , -1)
   - D (-1 , 0)
3. The distance between the two points (2 , 4) , (10 , -2) equal .....
   - A 9
   - B 10
   - C $3\sqrt{10}$
   - D 6
4. The circle $x^2 + y^2 = 25$ its centre (0 , 0) and passes through the point ....
   - A (1 , 4)
   - B (5 , 0)
   - C (25 , 0)
   - D (5 , 1)
5. The equation of a circle whose centre (3 , -5) and the length of its radius equal 7 units is:
   - A $(x - 3)^2 + (y - 5)^2 = 49$
   - B $(x + 3)^2 + (y + 5)^2 = 49$
   - C $(x + 3)^2 + (y - 5)^2 = 49$
   - D $(x - 3)^2 + (y + 5)^2 = 49$
6. The circumference of the circle whose equation $x^2 + y^2 = 8$ equals:
   - A 8π
   - B 64π
   - C $2\sqrt{2}\,\pi$
   - D $4\sqrt{2}\,\pi$

7) Write the equation of a circle whose centre M, and the length of its radius r where:

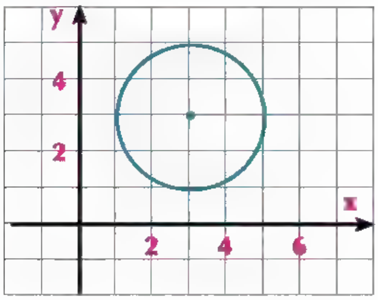
A M (2, 3) , r = 5 | **B** M(0 , 0) , r = 4

C M (3, 0) , r = 6 | **D** M(4 , -5) , $r = \sqrt{7}$

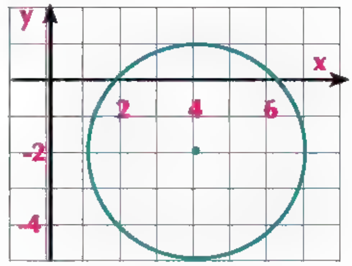
E M (0, -1) , $r = 2\sqrt{3}$ | **F** M(-4 , -3) , $r = \frac{3}{2}$

8) Write the equation of a circle represented by the given figure:

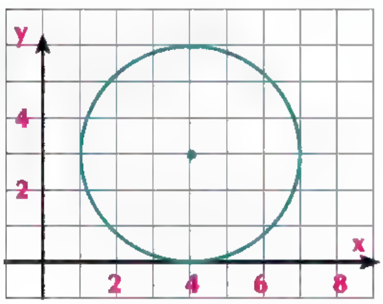
A



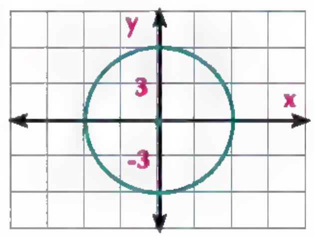
B



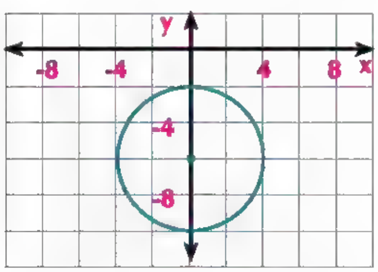
C



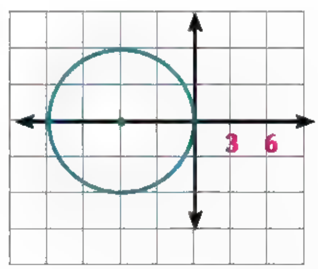
D



E



F



9) Find the equation of a circle if:

A Its centre M (7 , -5), and it passes through the point A(3 , 2).

B $\overline{AB}$ is a diameter in the circle, where A(6 , -4) , B(0 , 2).

C Its centre is the point (5 , -3) and touches the x-axis

10) **Find the centre, and the length of the radius for each of the following circles:**

A $x^2 + y^2 = 27$ | **B** $(x + 3)^2 + (y - 5)^2 = 49$

C $(x - 2)^2 + y^2 = 16$ | **D** $x^2 + (y + 7)^2 = 24$

11 **Write the general form of the equation of a circle in the following cases:**

A. Its centre M(3 , 1) ,and the length of its diameter equal 8.

B. Its centre M (0 , 0), and it passes through the point A (-1 , 3)

C. Its centre M (-5 , 0), and it passes through the point B (3 , 4)

D. $\overline{AB}$ is a diameter in it, where A(3 , -7) , B(5 , 1)

12 **Find the centre, and the length of the radius for each of the following circles:**

A. $x^2 + y^2 - 4x + 6y - 12 = 0$ B. $x^2 + y^2 + 2x = 8$

C. $x^2 + y^2 - 6x + 10y = 0$ D. $x^2 + y^2 - 8x = 12$

13 **Show which of the following circles are congruent:**

A. $x^2 + y^2 - 2x + 4y - 3 = 0$, $x^2 + y^2 + 6x - 11 = 0$

B. $x^2 + y^2 - 14x + 37 = 0$, $x^2 + y^2 + 10x + 13 = 0$

14 **Show which of the following equation is for a circle, then find its centre and the length of its radius:**

A. $x^2 + y^2 + 8x - 16y - 1 = 0$ B. $x^2 + 2y^2 + 6x - 5y = 0$

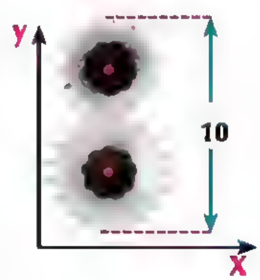
C. $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$ D. $x^2 + y^2 + 2xy - 12 = 0$

E. $x^2 + y^2 - 2x + 4y + 7 = 0$ F. $2x^2 + 2y^2 + 3y - 8 = 0$

15 **Marine navigation:** A radar is located in the position A(7 , -9), and cover a circular region. The length of its radius equals 30 length unit. Write the equation of the circle that determine the area of the radar range in the coordinates plane. Can the radar observe a ship in the position B (25 , -30)? Explain your answer.

16 **Architectural Design:** An architect designs a building in the form of a regular octagon. Its vertices passes by a circle $x^2 + y^2 - 4x + 12y - 60 = 0$. Calculate the area of the building to the nearest squared unit.

17 **Industry:** The opposite figure shows two gears in a machine. Their centres lie in a straight line parallel to the y- axis and the maximum distance between their edges is 10 units. Find the equation of the smallest gear, given that the equation of the bigger gear is $x^2 + y^2 - 10x - 8y + 32 = 0$.



18 **Creative thinking:** Find the equation of the circle which passes through the two points A(1 , 3), B(2 , -4) and its centre lies on the x-axis

General Exercises

For more exercises please visit the website of the Ministry of Education.

Summary of the unit

Axioms and concepts

The straight line: for any two points in the plane there is only one straight line passes through them.

The plane : is a surface with no ends such that a straight line joining any two points of its points lies wholly in its surface

The space: is the set of an infinite number of points, it contains all figures, planes and solids. the space contains four different non coplanar points at least.

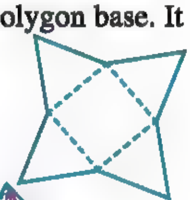
The relation between two different straight lines in the space: **1)** Intersected (if they intersected at only one point). **2)** Parallel: (if they lie in the same plane without intersection). **3)** Skew: (if they do not lie in the same plane (non-intersected and non-parallel).

The relation between a straight line and a plane in the space: **1)** The straight line cut the plane at a point. **2)** The straight line lies completely in the plane. **3)** The straight line does not intersect with the plane at any point in this case the plane is parallel to the straight line.

The relation between two different planes in the space: **1)** intersected at a straight line. **2)** Two parallel planes. **3)** Two coincided planes.

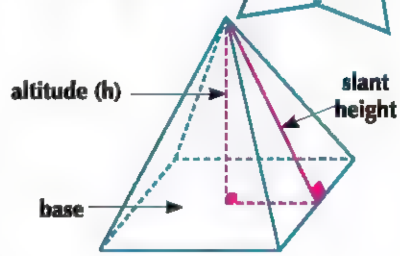
Net of a solid: It's a two dimensional figure that can be folded to be a three dimensional figure.

Pyramid: It's a solid which has a single base and all of its other faces are triangles that have a common vertex. The pyramid is named according to the number of sides of its polygon base. It could be a triangular pyramid, quadrangular pyramid, pentagonal pyramid .. and so on.



Regular pyramid : It's a pyramid whose base is a regular polygon. It's center is a foot of the perpendicular drawn from the vertex to the base and we find that:

- Its lateral edges are equal in length.
- Its lateral faces surfaces are isosceles and congruent triangles.
- The slant heights are equal.



The right pyramid: The pyramid is right if and only if, the foot of the perpendicular drawn from the vertex to its base is the geometrical center of the base.

The Right circular cone: It's the solid generated by the rotation of the right angled triangle a complete rotation around one of the sides of the right angle.

Lateral area of the pyramid $= \frac{1}{2}$ base perimeter × its slant height.

Total area of the pyramid = its lateral area + its base area.

Lateral area of the right cone $= \pi$ L r, where L is the length of the slant height, r is the length of its base radius.

Total area of the right cone $= \pi Lr + \pi r^2 = \pi r (L + r)$

Volume of pyramid equal one third the product of area of its base × its height.

Volume of cone equal one third the product of area of its base × its height

The circle: a set of plane points which are equidistant from a fixed point in the plane.

Equation of a circle The equation is a circle whose centre the point (d , h), and the length of its

radius equal r is: $(x - d)^2 + (y - h)^2 = r^2$

The general form of the equation of a circle its centre is the point (-L , -K), and the length of its radius r is: $x^2 + y^2 + 2Lx + 2ky + C = 0$, where $r = \sqrt{L^2 + k^2 - C}$, $L^2 + k^2 - C > 0$

To find the coordinates of the centre of the circle and the length of its radius from the general form of its equation:

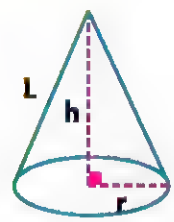
- First verify to put the equation in the general form where the coefficient of x^2 = coefficient of y^2 = unity.
- The coordinates of the centre $(-L , -k) = \left(\frac{-\text{ coefficient of } x}{2} , \frac{-\text{ coefficient of } y}{2}\right)$
- The length of the radius of the circle equal $r = \sqrt{L^2 + k^2 - C}$, where $L^2 + k^2 - C > 0$

Enrichment Information

Please visit the following links.

Accumulative test

Choose the correct answer:



1. All the following cases determine a plane except:
   - **A** A straight line and a point does not belong to it.
   - **B** Two parallel straight lines and not coincident.
   - **C** Two intersected straight lines.
   - **D** Two skew straight lines.

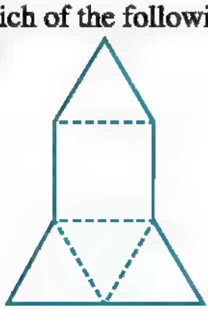
2. The total area for the right cone equal:
   - **A** πrL
   - **B** $\frac{\pi}{3} r^2h$
   - **C** $\pi r (r + L)$
   - **D** $\frac{\pi}{3} r (rh + 3L)$

3. A regular quadrangular pyramid, the perimeter of its base 36 cm, and its height 10 cm, then its volume equals .... cm^2
   - **A** 810
   - **B** 180
   - **C** 360
   - **D** 270

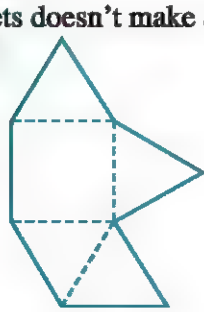
4. The center of the circle : $(x + 2)^2 + y^2 + 2y = 0$, is the point:
   - **A** (2 , 2)
   - **B** (-2 , -1)
   - **C** (2 , -1)
   - **D** (-4 , 2)

5. Which of the following nets doesn't make a regular quadrilateral pyramid when it folded?

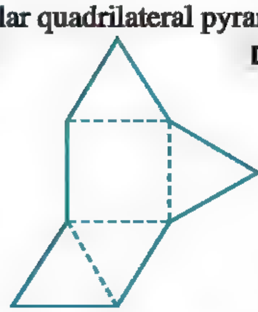
A



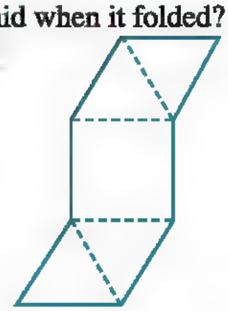
B



C



D



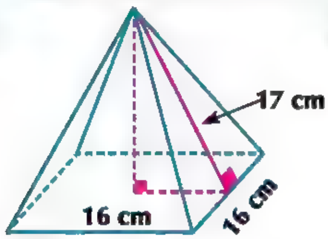
Questions with short answer

6. How many straight lines can be drawn in each of the following cases:
   - **A** Passes through two different points.
   - **B** Passes through three non-collinear points.
   - **C** Passes through two intersected planes.
   - **D** Passes through four points in the space, each three points from them are non-collinear.

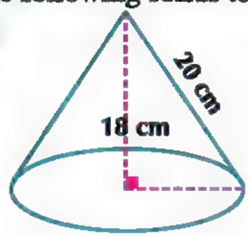
7. How many planes can be drawn passing through each of the following?
   - **A** A point **B** Two determine points **C** Three non-collinear points

8. 8 Find the volume of each of the following solids to the nearest cm^3.

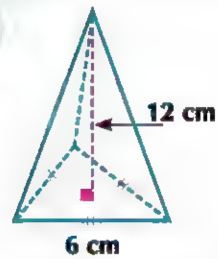
A



B



C



9. Find the equation of a circle, its center is the point (2 , -7) and its passes through the point (1 , 3).

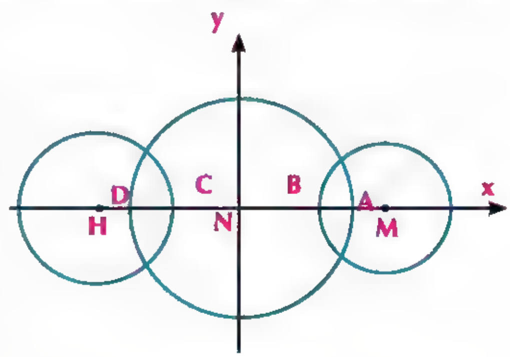
10. Which of the following circles are congruent? Explain your answer.
    - **A** $x^2 + y^2 + 4x - 2y - 5 = 0$, $x^2 + y^2 + 6x - 4 = 0$
    - **B** $x^2 + y^2 - 4x + 8y = 0$, $x^2 + y^2 + 12y + 16 = 0$

Questions with long answer:

11. Calculate to the nearest tenth, the volume of a regular pentagonal pyramid if the length of its polygon base side =16 cm and its height =12 cm.

12. Circular sector MAB, of radius 18 cm, and the measure of its central angle 60° it is folded and their radii are connected, to form the greatest lateral area of a right cone. Find the volume of this cone.

13. In the opposite figure: the points M, N, H lie on the x-axis of a coordinates perpendicular plane, N is the origin point if M, N, H are the centers of three circles the length of their radii are 5, 9, 6 of units respectively, CD = 2MA = 4 units. Find the general form of the equation of each of the three circles.



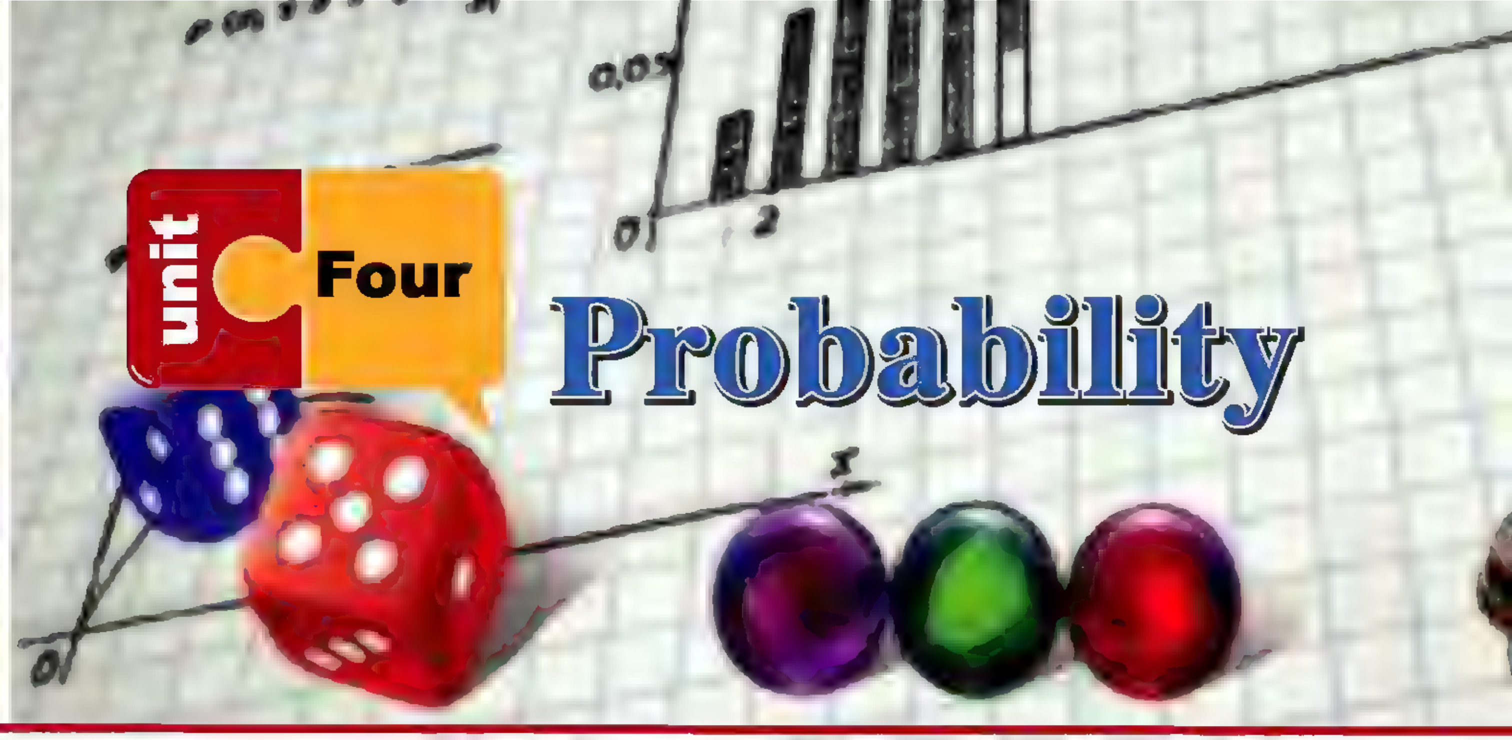
14. At a certain moment, a pilot recognizes two valleys met at a point and a street is passing over them without passing through their point of intersection. Represent this view by drawing,then determine the number of the formed planes.

Do you need an additional help:

If you can't solve the question no.	1	2	3	4	5	6	7	8	9	10
Refer to	3 -2	3 - 3	3 - 4	3 -1	3 - 3	3 - 4	3 - 4	3- 3	General skills	3 - 4

Unit Four

Probability



Introduction

The roots of the science of probability are dept. to the revolution age from the studies of the scientists to space science, chance games and their trails to understand and analyze of the appearance of some elements from a large set of another elements in which (Gerolame Cardano) had done in the sixteenth century. And what (Pierr de Fermat) and (Blaise Pascal) lived in the seventeenth century.

In the way of which the science of probability improved and progress many definitions of the probability appear ,some is simple and depend on the sensitive reorganization ,some depend on the experimental method and the idea of the proportional repeating for the event we need to select by repeating the experiment several times under a fixed conditions. Probability is used to measure the capability of the occurrence of a certain event.

Starting from the ninetieth century, the probability theorems was discovered which considered as the largest aid to the statistical work .the scientists (Laplace) is one of its establishers and also the scientists (Adolph Quteelet) who present the first statistical work in a scientific way at 1853 .starting from this date statistics and probability take their places as valued science ,its great value in different fields and also in the scientific researched in different fields.

It passes all of this by determine a correct calculations in which we can depend on in predicting in the coming aspects. In this unit we will deal with Basic terms and concepts in probability and how to calculate it.

Learning outcomes

By the end of the unit the students should be able to:

- Recognize the concept of the random experiment.
- Recognize the concept of the sample space.
- Write the sample space for some random experiments.
- Recognize the concept of the event, the simple event, the sure event and the impossible event.
- Recognize the concept of the mutually exclusive events.
- Recognize the operations on the events as (union, intersection ,difference ,complement)
- Recognize the concept of the probability.
- Use the axioms of probability to determine the probability of the occurrence of an event.
- Solve applied questions using the axioms of probability.
- Solve life applications problems using the laws of probability.

Essential key terms

- Statistics
- Probability
- Random Experiment
- Sample space
- Coin
- Die
- Event
- Simple Event
- Compound Event
- Certain Event
- Impossible Event
- Operation on the Events
- Mutually Exclusive Events

Lessons of the unit

Lesson (4 - 1): Calculating probability .

Materials

- Scientific calculator
- Graphical calculator
- Graphical programs

Chart of the unit

Propability

Random experiment

Sample space

event

certain | impossible | excpected | Mutually exclusive

Operation on the events

Intersction | union | complement | difference | De morgan's laws

Axioms of probability

laws of probability

Life Applications & problem solving

4 - 1 Calculating Probability

We will learn

- The concept of the random experiment and the sample space.
- The concept of the event –simple event-sure event-impossible event.
- Operations on events (union - intersection - difference - complement)
- Mutually exclusive events .
- De' Morgan's laws.
- Concepts of probability
- Calculating probability
- Probability axioms and its life applications.

Key - term

- random experiment
- sample space
- event
- simple event
- certain event
- impossible event
- mutually exclusive events
- probability
- probability axioms

Materials

- Scientific calculator.

Introduction:

In our previous study, we learned the probability in a simple way. So we will complete the study of these concepts and the operations on events while calculating the probability of the occurrence of an event through examples and different life applications.

Basic terms and concepts

Learn

The random experiment: It's an experiment we know all of its outcomes before we do it but we cannot predict which of these outcomes will occur when we do the experiment.

Example

1. Show which of the following experiments represent a random experiment?
   - **a** Rolling a regular die and observe the number written in its upper face.
   - **b** Draw a color ball from a bag including a set of color balls (without determine their color) and recognize the color of the drawn ball.
   - **c** Throw a coin and observe what appears in its upper face.
   - **d** Draw a ball from a bag including four balls identical in volume and weight. The first is white, the second is black, the third is red and the fourth is green. Recognize the color of the drawn ball.

Solution

The experiments (a),(c),(d) are random experiments because we know all of their outcomes before we do each of them but we cannot predict which of these outcomes will occur when we do these experiments.

The experiment (b) is not a random experiment because we cannot determine the outcomes of the experiment before we do it.

Try to solve

1. Show which of the following experiments represent a random experiment?

a Throw a coin twice and observe the sequence of heads and tails.

b Draw a numbered card from a bag contains a set of numbered cards (we do not know their numbers) and recognize the written number on the drawn card.

c Draw a card from a bag contains a set of 20 identical cards numbered from 1 to 20 and observe the written number on the drawn card.

Learn

definition

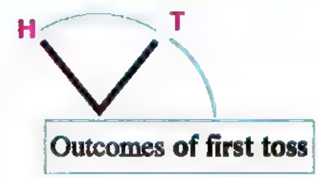
Sample space (outcomes space)

- The sample space of a random experiment is the set of all possible outcomes for this experiment, it denoted by (S)

Remarks:

- The number of elements in the sample space of a random experiment is denoted by n(S).
- The sample space will be finite if the number of its elements is finite and it will be infinite if the number of its elements is infinite. We will deal with the finite sample space.

The sample space for some famous random experiments:

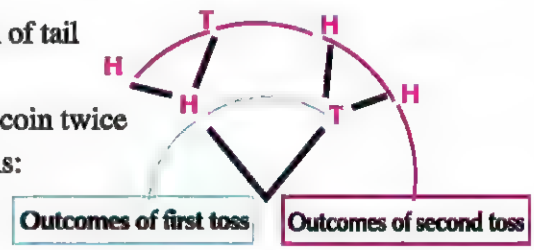


First: Tossing a coin

1- The sample space of the experiment of tossing a coin once and observing the shown face is : S = { H, T}

Where: H is the symbol of head, T is the symbol of tail

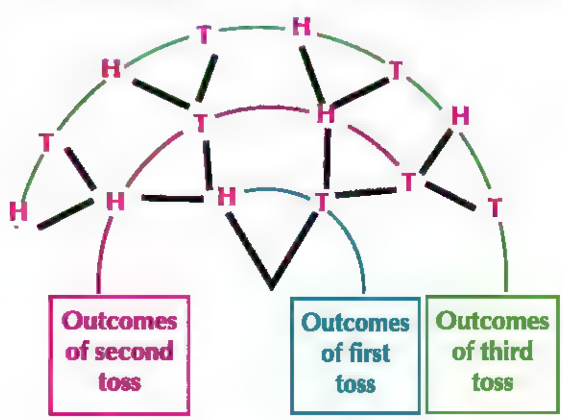
Where: n(S) = 2



2- The sample space of the experiment of tossing a coin twice and observing the sequence of heads, and tails is:

S = { (H, H), (H, T), (T, H), (T, T)}

Where: $n(S) = 2 \times 2 = 4 = 2^2$



3- The sample space of the experiment of tossing a coin three respective times and observing the sequence of heads and tails (could be as shown in the opposite tree diagram) is:

S = { (H , H , H) , (T , T , T) ,
(H , H , T) , (T , T , H),
(H , T , H) , (T , H , T),
(H , T , H) , (T , H , H)}

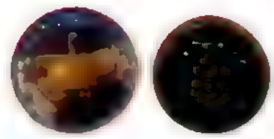
Where: $n(S) = 2 \times 2 \times 2 = 8 = 2^3$

Note that

1- On tossing a piece of coin m times, then $n(S) = 2^m$

2- (H, T) ≠ (T, H) why?

3- The sample space of the experiment of tossing two different coins (different in shape and volume) simultaneously (at the same time) is the same sample space of tossing a coin two successive times, and each result of the experiment results is written in the form of an ordered pair (the face of the 1^{st} coin, the face of the 2^{nd} coin).



Second: Rolling a die

1- The sample space of the experiment of rolling a die once and observing the number shown on the upper face is:

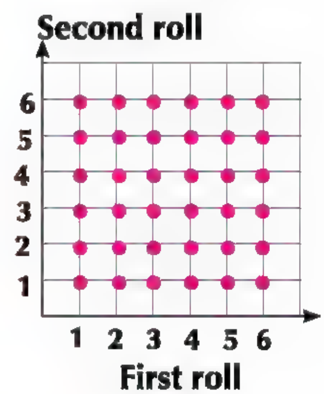
$S = \{ 1, 2, 3, 4, 5, 6 \}$ where: $n(S) = 6$

2- The sample space of the experiment of rolling a die two successive times and observing the number shown each time on the upper face is the group of ordered pairs. The first co-ordinate is the result of the first roll and the second co- ordinate is the result of the second roll. i.e.: $S = \{ (x, y) : x \in \{ 1, 2, 3, 4, 5, 6\}, y \in \{ 1, 2, 3, 4, 5, 6\}\}$ and the following figures illustrate this.

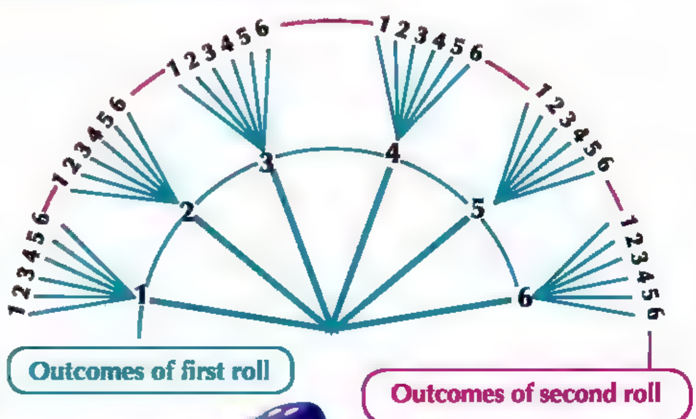
a Tabulated representation:

Toss First / Second	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b Geometrical representation:



c Tree diagram



Note that:

1- $n(S) = 6 \times 6 = 36 = 6^2$

2- $S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$

3- The sample space of the experiment of rolling two different dies at the same time (simultaneously) is the same sample space of rolling a die two successive times.

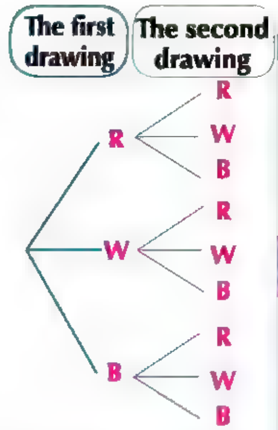
Example

(2) A bag contains three identical balls: the first is red, the second is white and the third is yellow. Write down the sample space if you draw two balls, one after the other, while reputing the drawn ball before drawing the other one (with replacement) and observing the succession of colors.

Solution

Denoting the red ball with the letter (R), the white ball with the letter (W) and the yellow ball with the letter (B):

First : When the ball is returned back to the bag before the second ball is drawn, each ball has the chance of appearance in the second drawing and it is then possible to draw the same ball twice. The opposite fig. shows the tree diagram of the sample space where n (S) = $3^2 = 9$



S
(R , R)
(R , W)
(R , B)
(W , R)
(W , W)
(W , B)
(B , R)
(B , W)
(B , B)

S = {(R, R), (R, W), (R, B), (W, R), (W, W), (W, B), (B, R), (B, W), (B, B)}

Add to your information

If a ball is drawn without replacement, that means: not to reputing the drawn ball before drawing the other one ,so there is no possible chance for that ball to appear in the second drawing.

Try to solve

(2) A box contains three identical balls numbered from 1 to 3. Two balls are drawn one after another with replacement and observing the number on the ball. Write down the sample space and find the number of its elements.

Learn

definition

The event

➤ An event is a subset from the sample space.

The simple event

➤ Is a subset from the sample space that contains only one element.

definition

The certain event

It is an event whose elements are the elements of the sample space S.
And it is an event that must occurs in each time we do the experiment.

The impossible event

It is an event that has no elements and is denoted by the symbol ϕ
And it is an event that must not occur each time we do the experiment.

Example

(3) In the experiment of throwing a coin several times, the experiment will stop if a head or three tails appear.

Write down the sample space, and then determine the following events:

A The appearance of head at most

B The appearance of head at least

C The appearance of two tails at least

D The appearance of two heads at least

Solution

From the drawing, we find:

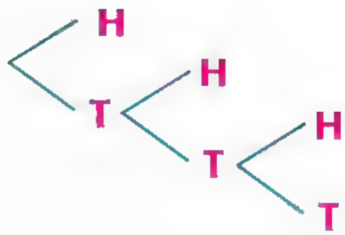
S = {H , (T , H),(T , T , H),(T , T , T)}

A = {H,(T , H),(T , T , H),(T , T , T)} = S

B = {H,(T , H),(T , T , H)}

C = {(T , T , H) , (T , T , T)}

D = {   } = ϕ the impossible event



Try to solve

(3) In the experiment of throwing a coin several times, the experiment will stop if two heads or two tails appear.

Write down the sample space, and then determine the following events:

A The appearance of head at least

B The appearance of two tails at most

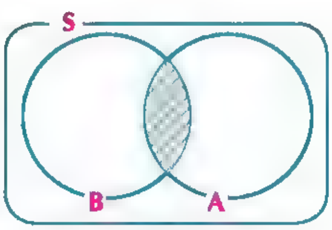
C The appearance of tail at most

Operation of the events

Learn

First: Intersection

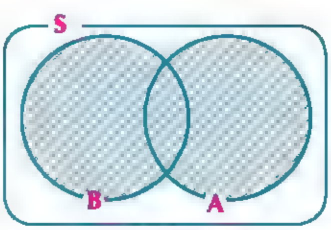
The intersection of the two events A and B is the event $A \cap B$ which contains all elements of the sample space that belong to both A and B and **means the occurrence of A and B (the occurrence of the two events at the same time).**



$A \cap B$

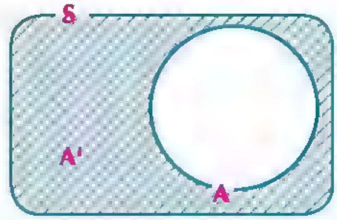
Second: Union

The union of the two events A and B is the event $A \cup B$ which contains all elements of the sample space that belong to A or B or both of them and **means the occurrence of A or B (the occurrence of one of them at least).**



$A \cup B$

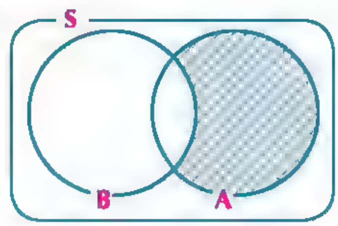
Third: Complement



The event A` is called the complementary event of the event A, where A` contains all elements of the sample space that does not belong to the event A, and **means non - occurrence of the event A**.

Note : $A \cup A' = S$, $A \cap A' = \phi$

Fourth: Difference



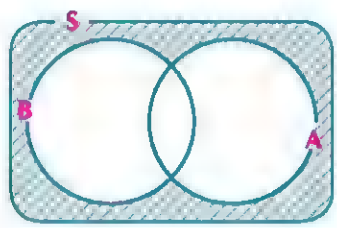
The event A - B contains all elements of the sample space that belong to A and does not belong to B it also contains the same elements of the event $A \cap B'$

and it means the occurrence **of A and non-occurrence of B (occurrence of A only).**

$A - B = A \cap B' = A - (A \cap B)$

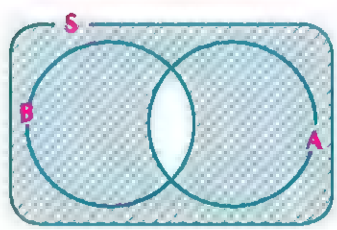
Fifth: De morgan's laws

A and B are the two events from the sample space S, then:



(first) $A' \cap B' = (A \cup B)'$

which means the event of (non-occurrence of any of the two events) or the event of (the non-occurrence of A and the non-occurrence of B)



(second) $A' \cup B' = (A \cap B)'$

which means the event of (non-occurrence of any of the two events all together) or the event of (the occurrence of one of the events at most)

Learn

Mutually exclusive events

Two events A and B are mutually exclusive events if the occurrence of one of them prevent the occurrence of the other

For example: **1-** If A" event of success in an exam" , B "event of failure in the same exam", then The occurrence of one of them prevent the occurrence of the other.

2- In experiment of rolling a die once, observing the number on the upper face then $S = \{1, 2, 3, 4, 5, 6\}$

If A: appearance of an odd number $A = \{1, 3, 5\}$

B: appearance of an even number $B = \{2, 4, 6\}$

then $A \cap B = \phi$ so, the occurrence of one of them prevent the occurrence of the other.

definition

- Two events A and B are said to be mutually exclusive if $A \cap B = \phi$
- Several events are said to be mutually exclusive if and only if each two by two are mutually exclusive events.

Notice that :

1- If $A \cap B = \phi$, then A and B are mutually exclusive events.
If A, B and C are three events in a sample space S and: $A \cap B = \phi$, $B \cap C = \phi$, $C \cap A = \phi$
then: A , B, C are said to be mutually exclusive events and vice versa.

2- Simple events (primary) in any random experiment are mutually exclusive.

3- Any event A and its complement A\` are mutually exclusive events.

Example

(4) Two distinct dice are tossed and observing the numbers on the upper faces.

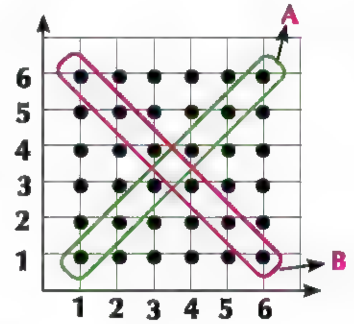
First: represent the sample space geometrically, and then write down the following two events.

A "appearance of the same numbers on the two faces"

B "appearance of two numbers their sum equals 7"

Second : Are A , B mutually exclusive? Justify your answer

Solution



First : The elements of the sample space are ordered pairs, their number = $6^2 = 36$

the opposite figure is the geometrical representation of the sample space where every element is represented by a point

A = {(1, 1), (2, 2), (3, 3), (4, 4) , (5, 5) , (6, 6) }

B = { (6, 1) , (5, 2), (4, 3) , (3, 4) , (2, 5) , (1, 6) }

Second: $\because A \cap B = \phi$ $\therefore$ A, B are mutually exclusive events

Try to solve

(4) In the previous example: write each of the following events:

C "appearance of two numbers their sum equal 5"

D "appearance of two numbers one of them is twice the other"

Are C , D mutually exclusive? Justify your answer.

Propability

Learn

Calculation of probability :

If A is an event in the sample space S for a random experiment all its outcomes (primary events) are equal possibility i.e. $A \subset S$, number of elements of event A equals n (A), number of elements of S equals n(S), if we denoted the probability of occurrence of A by P(A):

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{The number of the outcomes leads to the occurence of the event A}}{\text{The number of all possible outcomes for the random experiment}}$$

Example

5 If a ball drawn from a box contains 10 identical balls, 5 of them are white, 2 are red and the rest are green .Find the probability of the following events:

A the event "the drawn ball is red"

B the event "the drawn ball is red or green"

C the event "the drawn ball is not green"

Solution

The probability that the drawn ball is red $= P(A) = \frac{\text{The number of red balls}}{\text{The number of all balls}} = \frac{2}{10} = 0.2$

The probability that the drawn ball is red or green $= \frac{\text{The number of red balls + green balls}}{\text{The number of all balls}}$

$$= \frac{2+3}{10} = \frac{5}{10} = 0.5$$

The probability that the drawn ball is not green = P(C)

= The probability that the drawn ball is red or white $= \frac{2+5}{10} = 0.7$

Think: Can you obtain P(C) with another method? Explain that.

Try to solve

5 In the previous example: find the probability of the following events:

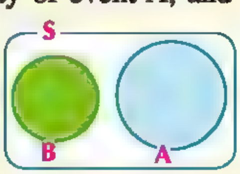
D: the event "the drawn ball is red or green"

E: the event "the drawn ball is red or white or green"

Learn

Axioms of probability

1- For every event $A \subset S$ there exists a real number called probability of event A, and denoted by P(A) Where : $0 \leqslant P(A) \leqslant 1$



2- $P(S) = 1$

3- If $A \subset S$, $B \subset S$

and A , B are mutually exclusive events, then : $P(A \cup B) = P(A) + P(B)$

From the previous axioms we notice that:

The first axiom means that the probability of the occurrence of any event is a real number belongs to the interval [0, 1]

The second axiom means that the probability of the sure event = 1

The third axiom is called the sum of probabilities of mutually exclusive events rule which is circulating for a finite number of mutually exclusive events.

$P(A_1 \cup A_2 \cup A_3 \cup ... \cup A_n) = P(A_1) + P(A_2) + P(A_3) + ... + P(A_n)$

where $A_1, A_2, A_3, ..., A_n$ are two by two mutually exclusive events

Important Corollaries

(1) $P(\phi) = 0$

(2) $P(A') = 1 - P(A)$

(3) $P(A - B) = P(A) - P(A \cap B)$

(4) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Add to your information

If $A \subset B$ then $P(A) \leqslant P(B)$

Example

(6) If A , B are two events in a sample space S of a random experiment, such that:

$P(A) = \frac{3}{8}$, $P(B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, Find:

a $P(A \cup B)$ b $P(A')$ c $P(A - B)$ d $P(A' \cap B')$

Solution

a $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{8} + \frac{3}{4} - \frac{1}{4} = \frac{7}{8}$

b $P(A') = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

c $P(A - B) = P(A) - P(A \cap B) = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

d $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{7}{8} = \frac{1}{8}$

Try to solve

(6) In the previous example, find the following probabilities:

a $P(B')$ b $P(B - A)$ c $P(A' \cup B')$

Example

(7) If A , B are two events in a sample space S of a random experiment, such that $P(A) = \frac{5}{8}$,

$P(B) = \frac{1}{2}$, $P(A - B) = \frac{3}{8}$ Find :

a $P(A \cap B)$ b $P(A \cup B)$ c $P(A' \cap B')$ d $P(A' \cup B)$

Solution

a $P(A \cap B) = P(A) - P(A - B) = \frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$

b $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{8} + \frac{1}{2} - \frac{1}{4} = \frac{7}{8}$

c $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{7}{8} = \frac{1}{8}$

d $P(A' \cup B) = P(A \cap B')' = 1 - P(A \cap B') = 1 - P(A - B)$

$= 1 - \frac{3}{8} = \frac{5}{8}$

Think : Can you obtain $P(A' \cup B)$ with another method? Explain that.

Try to solve

(7) In the previous question, find the following probabilities:

a) $P(A')$ b) $P(A' \cup B')$ c) $P(B \cap A')$

Example

(8) If A and B are two events in a sample space S of a random experiment, such that $P(A') = \frac{1}{3}P(A)$, $P(B) = \frac{1}{2}$, $P(A' \cup B') = \frac{5}{8}$ Find :

a The probability of occurrence of one of the two events at least.

b The probability of occurrence of one of the two events at most.

c The probability of occurrence of the event B only

d The probability of occurrence of only one of the two events.

Solution

$\because P(A' \cup B') = \frac{5}{8}$ $\therefore P(A \cap B)' = 1 - P(A \cap B) = \frac{5}{8}$ $\therefore P(A \cap B) = \frac{3}{8}$

$\because P(A') = \frac{1}{3}P(A)$ $\therefore 1 - P(A) = \frac{1}{3}P(A)$ $\therefore \frac{4}{3}P(A) = 1$ $\therefore P(A) = \frac{3}{4}$

a The probability of occurrence of one of the two events at least $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{3}{4} + \frac{1}{2} - \frac{3}{8} = \frac{7}{8}$

b The probability of occurrence of one of the two events at most $= P(A \cap B)'$

$= P(A' \cup B') = \frac{5}{8}$

c The probability of occurrence of the event B only $= P(B - A)$

$= P(B) - P(A \cap B) = \frac{1}{2} - \frac{3}{8} = \frac{1}{8}$

d The probability of occurrence of only one of the two events $= P(A \cup B) - P(A \cap B)$

$= \frac{7}{8} - \frac{3}{8} = \frac{1}{2}$

Think: Can you find the probability of occurrence of only one of the two events with another method? Explain this.

Try to solve

(8) If A and B are two events in a sample space S of a random experiment, such that $P(A) = 0.8$, $P(B) = 0.6$, $P(A \cup B)' = 0.1$ Find The probability of the following events:

a The occurrence of one of the two events at least.

b The occurrence of the event A only

c The occurrence of only one of the two events

d The occurrence of one of the two events at most.

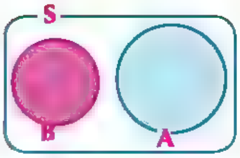
Example

(9) A and B are two events in a sample space S of a random experiment, where:

$P(B) = 3P(A)$, $P(A \cup B) = 0.72$, find: $P(A)$, $P(B)$

First: if A , B are mutually exclusive events. **Second:** if $A \subset B$

Solution

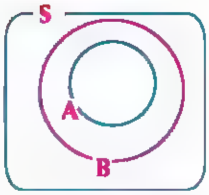


Let $P(A) = x$ $\therefore P(B) = 3x$

First: $\because$ A , B are mutually exclusive events.

$\therefore P(A \cup B) = P(A) + P(B)$ $: 0.72 = 3x + x$

$\therefore x = 0.18, P(A) = 0.18$, $P(B) = 0.54$



Second: $\because A \subset B$ $\therefore A \cup B = B$

$P(A \cup B) = P(B) = 3x = 0.72$

$\therefore P(A) = 0.24$, $P(B) = 0.72$

Try to solve

9. If A and B are two events in a sample space of a random experiment , Where:

$P(B) = \frac{1}{5}$, $P(A \cup B) = \frac{1}{3}$ Find P (A)

a If A , B are mutually exclusive events. **b** if $B \subset A$

Critical thinking:

Explain how to calculate P (A) if $A \subset S$, S is a sample space of a random experiment, if

$\frac{P(A')}{P(A)} = \frac{3}{7}$

Try to solve

10. IF S is a sample space of a random experiment where S = {A , B , C}, and $\frac{P(A')}{P(A)} = \frac{2}{3}$, $\frac{P(B')}{P(B)} = \frac{5}{2}$ Find P(C)

Example

10. **Join with the school inviroment:** If the probability of success of a student in the physics exam equals 0.85 and the probability of success in mathematics exam equals 0.9 and the probability of success in both subjects equals 0.8. Find the probability of:

a The success of the student in at least one of the two subjects.

b The success of the student in mathematics only.

c The non-success of the student in both subjects together.

Solution

Let A denoted the event of success of the student in physics , B: denoted the event of success of the student in mathematics.

then: $P(A) = 0.85$, $P(B) = 0.9$, $P(A \cap B) = 0.8$

a Probability of success of the student in at least one of the two subject = $P(A \cup B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.85 + 0.9 - 0.8 = 0.95$

b Probability of success of the student in mathematics only means probability of success in mathematics and not success in physics $P(B - A)$

$\therefore P(B - A) = P(B) - P(B \cap A) = 0.9 - 0.8 = 0.1$

c. The event of the student will not success in both subjects = $(A \cap B)'$ which is the complement of $(A \cap B)$

$\therefore P(A \cap B)' = 1 - P(A \cap B) = 1 - 0.8 = 0.2$

Life applications:

Try to solve

(11) To get a job in a company, a person has to pass two exams: theoretical and practical. If the probability to succeed in the theoretical exam is 0.75, the probability to succeed in the practical exam is 0.6 and the probability to succeed in both of them is 0.5. If a person applies to this job for the first time. Find the probability of:

a Success in the theoretical exam only.

b Success in at least one of the two exams.

Critical thinking:

Join with sport: A coach of one of the sports teams says in a news briefing that the probability that his team wins in the away match is (0.7), the probability of winning in the rematch is (0.9) and the probability of winning both matches is 0.5. Does the concept of probability agree with the words of the coach? Justify your answer.

Example

(11) A die is rolled two consecutive time. The number on the upper face is observed in each time. Determine each of the following events:

First : A The appearance of two numbers their sum is less than or equal 4.

Second: B One of the two numbers is twice the other.

Third: C The asbsolute difference between the two numbers equals 2

Fourth: D The appearance of two numbers their sum is more than 12

Solution

$n(S) = 36$

First: $A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$ $\therefore n(A) = 6$, $\therefore P(A) = \frac{6}{36} = \frac{1}{6}$

Second: $B = \{(1, 2), (2, 1), (2, 4), (4, 2), (3, 6), (6, 3)\}$ $\therefore n(B) = 6$ $\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

Third: $C = \{(1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4)\}$ $\therefore P(C) = \frac{8}{36} = \frac{2}{9}$

Fourth: Its impossible to get two numbers their sum is more than 12, $\therefore D = \phi$, $P(D) = 0$

Try to solve

(12) In the previous example, calculate the following probabilities:

First: A the event "the two appearance numbers are equal".

Second: B the event "the number in the first roll is odd and the number in the second roll is even"

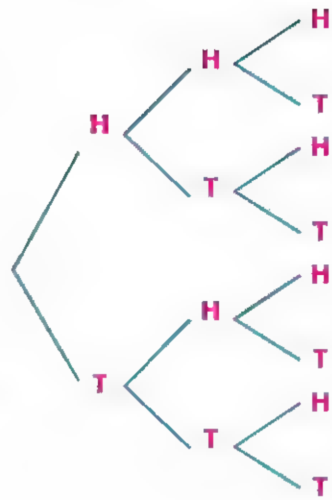
Example

(12) A coin is tossed three consecutive times, the sequence of heads and tails is observed. Find the probability of each of the following:

First: A appearance of only one head.

Second: B appearance of at least two heads.

Third: C appearance of exactly two heads.



Solution

$S = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T),$
$(T, H, H), (T, H, T), (T, T, H), (T, T, T)\},$
$n(S) = 8$

First: $\because$ A appearance of only one head.

$\therefore A = \{(H, T, T), (T, H, T), (T, T, H)\},$

$\because n(A) = 3 \qquad \therefore P(A) = \frac{3}{8}$

Second: $\because$ B appearance of at least two heads means either two or more heads

$\therefore B = \{(H, H, T), (H, T, H), (T, H, H), (H, H, H)\}$

$\because n(B) = 4 \qquad \therefore P(B) = \frac{4}{8} = \frac{1}{2}$

Third: $\because$ C appearance of exactly two heads

$\therefore C = \{(H, H, T), (H, T, H), (T, H, H)\} \quad \because n(C) = 3 \qquad \therefore P(C) = \frac{3}{8}$

Try to solve

(13) In the previous example, calculate the probability of:

First: A appearance of the same face in the three tosses **Second:** B appearance of at most one head.

Third: C appearance of odd number of heads **Fourth:** D appearance of at least one tail.

Fifth: E appearance of number of heads equals number of tails.

Example

(13) **Join with community:** In one of the conferences, 200 persons from different nationalities participated in the conference and their data is represented by the following table:

	Speak Arabic	Speak English	Speak French	Total sum
[illegible]	50	45	25	120
[illegible]	45	30	5	80
[illegible]	95	75	30	200

If one of the participants is chosen randomly, then find the probability that the chosen person is:

a A woman speaks Arabic.
b A man speaks English.
c Speaks Arabic or French.
d Speaks Arabic and English.
e A woman does not speak English and does not speak Arabic.

Solution

a The probability that the chosen person" A woman speaks Arabic" $= \frac{45}{200} = 0.225$

b The probability that the chosen person" A man speaks English" $= \frac{45}{200} = 0.225$

c The probability that the chosen person" Speaks Arabic or French" $= \frac{95+30}{200} = 0.625$

d The probability that the chosen person" Speaks Arabic and English" $= p(\phi) = 0$

e The probability that the chosen person" A woman does not speak English and does not speak Arabic" $= \frac{5}{200} = 0.025$

Try to solve

(14) In the previous example, find the probability that the chosen person:

a Does not speak English.
b Speaks German.
c A woman speaks French or English.
d A man speaks Arabic or a woman speaks English.

Exercises (4 - 1)

(1) A student wants to buy a bag. It is possible to choose from three types. Each one has two sizes and the color of the bag is either black or brown. Represent the sample space by a tree diagram.

(2) In an experiment of tossing a coin once, then a die is rolled, observing the upper faces.

a Write down the sample space of this experiment, then determine the following events.

- A: appearance of a head and an odd number.
- B: appearance of a tail and an even number
- C: appearance of a prime number > 2
- D: appearance of a number divisible by 3

(3) A die is rolled two consecutive times, the number on the upper face is observed in each time Determine each of the following events:

- A: The appearance of two equal numbers
- B: The appearance of two numbers their sum equals 9
- C: The appearance of two numbers their sum equals 13
- D: The appearance of number 3 one time at least.

(4) From the set of numbers $\{1, 2, 3, 4\}$ we need to form a two different digit number. Represent the sample space in a tree diagram, and then determine the following events:

- A: The event "the unit digit is an odd number ".
- B: The event "the tens digit is an odd number".
- C The event "the two digits are odd numbers".
- D The event "the unit digit or the tens digit is an odd number.

5. A bag contains 20 identical cards numbered from 1 to 20, If a card is selected randomly and the number written on it is recorded. Write the following events:
   A the event "the recorded number is even and greater than 10
   B the event "the recorded number is a factor of 12"
   C the event "the recorded number is odd and divisible by 3
   D the event "the recorded number is a multiple of the two numbers 2, 5
   E the event "the recorded number is prime"
   F the event "the recorded number satisfying the inequality $5x - 3 \leqslant 17$
6. Two cards are drawn one after the other from a set of 8 identical cards numbered from 1 to 8 and the drawn card must returned before drawn another card. What is the number of the elements in the sample space? and if:
   A: is the event "the number in the second draw is three times the number in the first draw"
   B: is the event "the sum of the two numbers is more than 13"
   Write the events A, B. Are there two mutually exclusive events? Explain that.
7. In the experiment of tossing a coin three consecutive times and observing the sequence of heads and tails .represent the sample space with tree diagram, then determine the following events:
   A the event "appearance of two tails at least" B the event "appearance of two tails at most"
   C the event " appearance of a head in the first toss"
   A the event " non-appearance of a head in the three tosses"
8. In an experiment of tossing a coin once, then a die is rolled, observing the upper faces Represent the sample space of this experiment by a probability tree diagram, and then determine the following events:
   A appearance of a Tail and an even number"
   B appearance of a head and an odd number"
   C non-occurrence of A or non- occurrence of B"
   D occurrence of the event A only
   E occurrence of the event A and occurrence of the event B

Choose the correct answer from those given :

9. If a regular die is rolled once, then the probability of the appearance of an odd number less than 5 in the upper face equals:
   (a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$
10. If a regular die is rolled twice, then the probability of the appearance of an even number in the first roll and a prime number in the second roll equals:
   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{4}$
11. If a ball is drawn randomly from a box contained 3 white balls, 5 red balls and 7 green balls, then the probability that the selected ball is white or green equals:
   (a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{7}{15}$ (d) $\frac{1}{2}$
12. A card is drawn from a set of 9 identical cards numbered from 1 to 9. What is the probability that the drawn card carrying a divisor of (factor of) 9 or an odd number equals:

a $\frac{1}{3}$ b $\frac{7}{9}$ c $\frac{1}{2}$ d $\frac{5}{9}$

13. If A , B are two events in a sample space of a random experiment $B \subset A$, $P(A) = 2P(B) = 0.6$ then $P(A - B)$ equals:

a 0.6 b 0.3 c 0.4 d 0.2

14. A uniform die, the numbers 8, 9, 10, 11, 12, 13 written in its faces. If the die is rolled once, observing the number appearing on its upper face

a Find the probability of each of the following events:

- A: "appearance of an odd number."
- B "appearance of a prime number."
- C: "appearance of an even number."
- D "appearance of a number great than 12."
- E: "appearance of a number consists of two digits."
- F "appearance of a number consists of only one digit."

b Calculate: $P(A \cup C)$, $P(E \cup F)$, $P(B \cap D)$.

15. If is a sample space of a random experiment, where S = {A , B, C, D}, find: P(A) , P(B) , given that $P(A) = 3P(B)$, $P(C) = P(D) = \frac{7}{18}$

16. If A, B are two mutually exclusive events, S is a sample space of its random experiment, If $P(A \cup B) = 0.6$, $P(A - B) = 0.25$ find, P(A), P(B).

17. If A, B is a sample space of a random experiment, and $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$ find:

a $P(A')$ b $P(A \cup B)$ c $P(A - B)$ d $P(A' \cap B')$

18. If A, B are two events, of a sample space of a random experiment, where: $P(A) = 0.4$, $P(B') = 3P(B)$, $P(A \cap B) = 0.2$ find the probability of:

a Occurrence of A only. b Occurrence of A or B

c Occurrence of A and non-Occurrence of B.

19. A box contains colored identical balls: 4 red, 6 blue, and 5 yellow. A ball is selected at random from the box, find the probability that the drawn ball is:

a Red. b Blue or yellow. c Not blue. d Not red and not yellow.

20. One card is selected at random from 30 identical cards numbered from 1 to 30. Find the probability that the selected card is carring a number:

a Divisible by 3 b divisible by 5

c Divisible by 3 and 5 d divisible by 3 or 5

21. Three distinct coins are tossed once. Observing the upper faces, find the probability of the following events:

- A: appearance of a head or two heads.
- B: appearance of at least one head.
- C: appearance of a head at most.
- D: appearance of at two consecutive tails at least.

(22) A die is rolled two consecutive times, the number on the upper face is observed in each time. Find the probability of each of the following events:

- Appearance of the number 4 in the first roll.
- Appearance of two numbers, their sum equals 8
- Appearance of two numbers, their sum is less than or equal 5

(23) **Join with sport :** A random sample consists of 60 persons in a survey, it is found that 40 of them encourage Al Hilal club, 28 of encourage El negma club and 8 of them don't encourage any of them. A person is chosen at random from the sample. Find the probability that the chosen person encourages:

a At least one of the two clubs.
b Both clubs.
c Al Hilal club only.
d Only one of the two clubs.

(24) In an experiment of tossing a coin once, then a die is rolled once , observing the upper faces. If A is the event of the appearance of a head and a prime number, B is the event of the appearance of an even number. Find the probability of the occurrence of each of the two events, and then calculate the probability of the following events:

a The occurrence of one of the events at least
b The occurrence of the two events together
c The occurrence of only the event B
d The occurrence of only one of the two events

(25) a card is selected randomly from 50 identical cards numbered from 1 to 50, if the number written on it is recorded. Find the probability that the number written on the selected card is:

a A multiple of number 7
b A perfect square number
c A multiple of number 7 and a perfect square number
d Not a perfect square number and not a multiple of 7

(26) If A , B are two events, in a sample space of a random experiment, where: $P(B) = \frac{4}{5} P(A)$, $P(A - B) = 0.24$ $P(B \cap A') = 0.15$ then find: $P(A)$, $P(B)$, $P(A \cup B)$, $P(A' \cup B')$

(27) Tarek wrote 75 letters on the typewriter ,he found that 60% of them are without mistakes and Zead wrote 25 letters on the typewriter ,he found that 80% of them are without mistakes. If a letter is selected randomly from all letters written by both Tarek and Zead ,then find the probability that the selected letter is:

a Without mistakes.
b Written by Zead.
c Written by Zead without mistakes.
d Written by Tarek with mistakes.

(28) If A , B are two events, in a sample space of a random experiment, where: $P(A) = 0.6$, $P(B) = 0.8$, $P(A' \cup B') = 0.5$ then find $P(A' \cap B)$

General Exercises

For more exercises please visit the website of the Ministry of Education.

unit Summary

1 **Random experiment:** It is an experiment in which we can specify in advance all its possible outcomes before carrying it, but we cannot predict in certainty which of these outcomes will exactly occur. When we do it.

2 **Sample space "Outcomes space":** It is the set of all possible outcomes of a random experiment. it is denoted by S

3 **An event :** It is a subset of the sample space.

4 **Simple event:** It is a subset of a sample space that contains a single element.

5 **Certain " sure" event:** It is the event whose elements are all of the sample space (S).

6 **Impossible event:** An event has no element , denoted by (ϕ).

7 **Operations on events:** Intersection - union - complement - difference.

8 **Mutually exclusive events**

- Two events A and B are mutually exclusive if $A \cap B = \phi$.
- Several events are said to be mutually exclusive events if each two by two events are mutually exclusive

9 **Calculations of probabilities**

- If S is the sample space of a random experiment ,the prime events on it are equal opportunities
- The probability of the occurrence of the event $A \subset S$ is denoted by P(A) such that $P(A) = \frac{n(A)}{n(S)}$

10 **Axioms of probability**

- For every event $A \subset S$ there exists a real number which is called probability of A and denoted by P where: $0 \leqslant P(A) \leqslant 1$
- $P(S) = 1$
- If $A \subset S$, $B \subset S$ and A, B are mutually exclusive events, then: $P(A \cup B) = P(A) + P(B)$

11 If $A_1 \cup A_2 \cup A_3 \cup ....\cup A_n = S$ where $A_1, A_2, A_3, ..., A_n$ are mutually exclusive events

then $P(A_1) + P(A_2) + P(A_3) + ... + P(A_n) = 1$

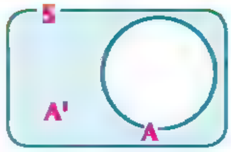
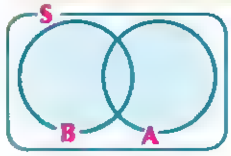
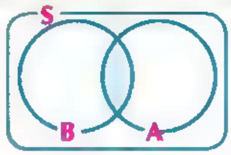
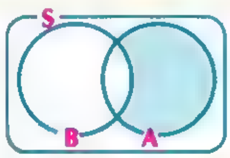
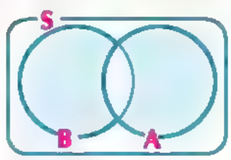
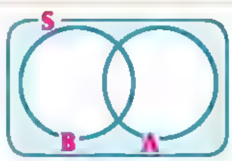
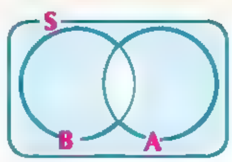
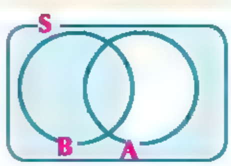
12 $P(\phi) = 0$

13 If $A \subset S$ where S is the sample space of a random experiment, then $P(A') = 1 - P(A)$

14 If A , B are two events in the sample space of a random experiment ,then:

a $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **b** $P(A - B) = P(A) - P(A \cap B)$

15 Events in a verbal form, their representations by venn diagram and their probabilities:

The event in a verbal description	Representation of the event by a Venn-diagram	Probability of occurrence of the event
Non occurrence of event A		$P(A') = 1 - P(A)$
Occurrence of A or B (occurrence of at least one of them)	 $A \cup B$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Occurrence of A and B (occurrence of both of them together)	 $A \cap B$	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
Occurrence of A only (occurrence of A and non occurrence of B)	 $A - B = A \cap B' = A - (A \cap B)$	$P(A - B) = P(A) - P(A \cap B) = P(A \cap B')$
Occurrence of only one of them (occurrence of A only or occ. of B only)	 $(A - B) \cup (B - A)$	$P((A - B) \cup (B - A))$ $= P(A - B) + P(B - A)$ $= P(A \cup B) - P(A \cap B)$
Non occ. of any of the two events. (Non occ. of event A and Non occ. of event B	 $(A \cup B)' = A' \cap B'$	$P(A \cup B)' = P(A' \cap B') = 1 - P(A \cup B)$
Non occ. of both of them together. (Non occ. of event A or Non occ. of event B) (The occ. of one of them at most)	 $(A \cap B)' = A' \cup B'$	$P(A \cap B)' = P(A' \cup B') = 1 - P(A \cap B)$
Non occurrence of A only (occ. of B or not occ. of A)	 $(A - B)' = B \cup A'$	$P(A - B)' = 1 - P(A - B) = P(B \cup A')$ $= P(A') + P(A \cap B)$

Accumulative Test

Complete the following:

1. In an experiment of rolling a die once and observing the number appearing on the upper face, then the sample space S = ...
2. In an experiment of tossing a coin twice and observing the upper face, then the event of the occurrence of a head at most = ...
3. In an experiment of rolling a die, then tossing a coin once and observing the upper face in each of them ,then the event of the appearance of a prime number = ...
4. In an experiment of rolling a die twice and observing the number appearing on the upper face each time, then the event that" the sum of the two numbers equals 5" = ...
5. If a card is drawn randomly from 20 identical cards numbered from 1 to 20, and its number is observed, then the event that" the appearance number is divisible by 3" = ...
6. In an experiment of tossing a coin three consecutive times and observing the appearance of the heads and the tails on its upper face, then the event of the appearance of "exactly two heads = ...
7. If $A \subset S$ where S is the sample space of a random experiment and $P(A') = 3\,P(A)$ find $P(A')$.
8. If a card is drawn randomly from 20 identical cards numbered from 1 to 20, and its number is observed, find the probability that the number on the drawn card:

   a Divisible by 6 b Prime number more than 10 c A factor of number 12

9. A and B are two events in a sample space for a random experiment, if $P(A \cup B) = 0.85$, $P(A) = 0.75$, $P(B') = 0.6$ find:

   a $P(A \cap B)$ b $P(A \cap B')$ c $P(A' \cup B')$

10. A and B are two events in a sample space for a random experiment, if $P(A) = \frac{2}{3} P(B)$, the probability of the occurrence of one of them at most equals 0.75 , the probability of the occurrence of one of them at least equals 0.6 find the probability of the following events:
    - a The occurrence of both of them all together.
    - b The occurrence of only one of them.
    - c The occurrence of B or the non-occurrence of A.
11. A and B are two events in a sample space for a random experiment, if $P(A') = \frac{3}{5} P(A \cup B) = 0.45$ find P (B) in the following cases:

    a A and B are mutually exclusive events b $A \subset B$ c $P(B - A) = 0.2$

12 **Join with Tourism:** A tour consists of 19 tourists from Russia, 17 from italy, 14 tourists from France. One of them was chosen at random, calculate the probability that the tourist:

- **a** From Russia or France.
- **b** Not from France.
- **c** From Europe.
- **d** From Netherlands.

13 **Join with school environment :** In a school celebration for exceling students, if the probability that the governor attends is 0.8, and the probability that the General Director of Education attends is 0.9 and the probability that both of them attend together is 0.75 find:

- **a** The probability that the governor only attends.
- **b** The probability that at least one of them attends.
- **c** The probability that both of them will not attend.

14 A and B are two events in a sample space S of a random experiment. If $P(A) = 0.6$, $P(B') = 0.3$, $P(A \cup B) = 0.9$, find the probability of each of the following events :

- **a** Occurrence of A or B
- **b** Occurrence of A and non-occurrence of B
- **c** Occurrence of A only or occurrence of B only

15 S is the sample space of a random experiment ,where $S = \{A, B, C\}$

if $\frac{P(A')}{P(A)} = \frac{7}{3}$, $2\,P(B) = 3\,P(B')$, Find $\frac{P(C')}{P(C)}$

16 If A, B are two events in a sample space S of a random experiment, and $P(A) = 0.6$, $P(B) = 0.5$, $P(A' \cup B') = 0.7$ Find: The probability of the following:

First: the occurrence of both of the two events together

Second: the occurrence of event A only

Third: the occurrence of at least one of the two events

Fourth: the occurrence of only one of the two events

17 50 persons request a job in one of the banks and their data is represented in the following table. If a person is selected randomly, find the probability that the selected person is:

First: female.

Second: With middle qualification.

Third: male with high qualification.

Fourth: female or with high qualification.

Gender	High qualification	Middle qualification	Sum
Male	16	14	30
Female	12	8	20
Sum	28	22	50

General Tests

Test One Mathematics Applications

Answer all of the following questions:

First Question: choose the correct answer from those given:

1. Two forces of magnitude 3F , 2F and the magnitude of their resultant is 5F, then the measure of the angle enclosed between the two forces equals:
   a. zero° b. 60° c. 20° d. 180°
2. If a car moved with a uniform velocity of magnitude 90 km/h for 30 min ,then the covered distance during this period of time measured by kilometer equals :
   a. $\frac{3}{4}$ b. 2.7 c. 45 d. 162
3. All of the following cases form a plane except:
   a. A straight line and a point do not belong to it.
   b. Two different parallel straight lines c. Two intersected straight lines
   d. Two skew straight lines
4. If a coin is tossed once on a horizontal surface ,and its upper face is observed ,then the probability of "non-appearance of a head " equals:
   a. zero b. $\frac{1}{3}$ c. $\frac{1}{2}$ d. 1

Second Question:

1. If the three coplanar forces $\overrightarrow{F_1} = 5\overrightarrow{i} + 3\overrightarrow{j}$, $\overrightarrow{F_2} = a\overrightarrow{i} + 6\overrightarrow{j}$, $\overrightarrow{F_3} = -14\overrightarrow{i} + b\overrightarrow{j}$ act at a point and their resultant $\overrightarrow{R} = (10\sqrt{2}, \frac{3}{4}\pi)$ find the values of a and b.
2. A body of weight 300 gm.wt is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$ The body is prevented from sliding by a force form with the line of the greatest slope an angle of measure 30° upwards. Find the magnitude of the force and the reaction of the plane.

Third Question:

1. A cyclist covered a distance 37.5 km in a road with velocity 25 km/h ,then he covered 18 km with velocity 12 km/h .find the magnitude and the direction of the average velocity vector during the whole trip if:
   a. The two displacements are in the same direction.
   b. The two displacements are in different directions.
2. A body moved in a straight line with uniform acceleration 5 cm/sec^2 and with an initial velocity 12 cm/sec in the opposite direction of its acceleration .find its velocity and its acceleration after :
   a. 3 sec. b. 4 sec. c. 6 sec. d. 9 sec.

Fourth Question:

1. If A, B are two events in the sample space of a random experiment and $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$ calculate: a. $P(A \cup B)$ b. $P(A \cup B')$

2. Find the general form for the equation of a circle whose center (2, - 1) and the length of its radius is 3cm.

Fifth Question:

1. A uniform smooth sphere of weight 10 gm. wt and radius length 30 cm is hanged from a point on its surface by a light strings of length 30 cm and the other end of the strings is fixed in a point on a vertical smooth wall. Find in the case of equilibrium each of:
   a) The tension on the string. b) The reaction of the wall on the sphere.
2. If the lengths of the radii of the Moon and the Earth are 1600 km, 6400 km respectively, and the ratio between the gravitational acceleration for each of them is 1:6 .Find the ratio between their masses respectively.

Test Two Mathematics Applications

Answer all of the following questions:

First Question: choose the correct answer from those given:

1. If a letter is chosen randomly from a set of letters S = { A,B,T,D,H,O,I,K,N,Z}, then the probability that the chosen letter is one of the letters of the word "THINK" is:
   a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$
2. The point that lies on the circle $(x - 2)^2 + y^2 = 13$
   a) (2, 3) b) (3, -2) c) (2, 5) d) (4, 3)
3. Two forces of magnitude 5 , 3 newton and the measure of the angle enclosed between them is $60°$,then the magnitude of their resultant R equals:
   a) 2 b) 7 c) 8 d) 5
4. 180 m/h/sec = ... cm/sec^2
   a) $\frac{1}{20}$ b) 5 c) 30 d) 300

Second Question:

1. A cube of wax with edge length 30 cm transfer into a right circular cone of height 45 cm. Find the length of the radius of the base of the cone, if 8% of the wax loss during milting and transferring processes.
2. If A , B are two events in the sample space of a random experiment and $P(A') = \frac{3}{8}$, $P(A \cup B) = \frac{3}{4}$ find P(B) in each of the following cases
   a) A , B are mutually exclusive events b) $P(A \cap B) = \frac{1}{8}$

Third Question:

1. A uniform rod of length 100 cm and weight 150 gm. wt is hanged freely by two strings and the other ends of the strings are fixed in one point. If the lengths of the two strings are 80 cm, 60 cm, find the tension in the two strings.

2. ABCDEF is a uniform hexagon, the forces of magnitudes 8, $6\sqrt{3}$, 5 and $4\sqrt{3}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ respectively. find the magnitude and the direction of their resultant.

Fourth Question:

1. The velocity of a car is reduced uniformly from 66 m/sec to 79.2 km/h during covering a distance of 66 m. Find the time established to cover this distance and the covered distance till it became at rest.

2. A body is projected vertically up from a point on the surface of the ground to return to it after 10 sec from the instance of projection, find:
   a. The initial velocity
   b. The maximum height the body can reach.

Fifth Question:

1. $\overline{AB}$ is a uniform rod with length 40 cm and weight 30 newton is attached with a vertical wall by a hang at A ,the rod is kept in equilibrium by a means of a light string connected by its ends with the rod at B and with the vertical wall at the point C above A by 40 cm. Find the magnitude of the tension in the string and the reaction of the hang at A.

2. A particle moves such that its position vector $\vec{r}$ is given as a function of time in term of the two fundamental unit vectors $\vec{i}$, $\vec{j}$ by the relation $\vec{r} = (6t - 3)\ \vec{i} + (8t + 1)\ \vec{j}$. Find the magnitude of the displacement vector at t = 3.